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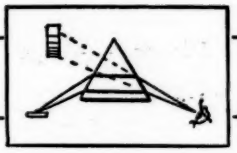

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Journal of the
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VOLUME 28 DECEMBER, 1938 NUMBER 12

The Blackening of Photographic Plates by Long Wave-Length X-Rays, II

F. R. HIRSH, JR.*
Cornell University, Ithaca, New York
(Received September 29, 1938)

Density-exposure time curves in the region of 11.0 to 14.0A are presented. A new linear relation between density and exposure time is shown which may prove useful to photographic workers. An empirical expression is obtained connecting density and exposure time.

INTRODUCTION

THE author has for some time been interested in studying the curves of photographic density, D, *vs.* exposure time, t (x-ray intensity I_c constant) for long wave-length x-rays. Curves[1] of density *vs.* exposure time for several soft x-ray wave-lengths were presented in 1935. The paper extends this study to longer wave-lengths.

EXPERIMENTAL

In Fig. 1(a) are shown curves of density *vs.* exposure time for x-rays of wave-lengths ~11.0 to 14.0A, secured by the method described in the previous paper.[1] $L\beta_1$ line radiation was used in a Siegbahn vacuum spectrograph to expose Eastman IV–O plates. The density at each line maximum was measured by means of an auxiliary magnifying scale on the recording drum box of a Moll microphotometer. Each maximum line density was corrected for background. The corrected maximum line density was plotted against exposure time.

The curves in Fig. 1(a) from 12.0 to 14.0A exhibit a peculiar shape, different from similar curves for shorter wave-lengths. The successive lowering of each curve in Fig. 1(a) may be due to increasing absorption in path and target, with increasing wave-length (each curve was taken at 20 kv and 5 ma).

INTERPRETATION

The author, for certain reasons, tried various ways of plotting density against exposure time. Besides D *vs.* t of Fig. 1(a), D *vs.* $\log_{10} t$ and $\log_{10} D$ *vs.* $\log_{10} t$ were tried. In Fig. 1(b), $\log_{10} D$ is plotted against $\log_{10} t$, using the data of Fig. 1(a). In each case a considerable linear region results. The lower three curves show a break at approximately $\log_{10} t \sim 2.0$.

Encouraged by this result, the writer replotted the data given in reference (1) as $\log_{10} D$ against $\log_{10} t$; again a considerable linear region resulted in each curve. See Fig. 2.

This linear relation holds in the ultraviolet region also.* See Figs. 3(a) to (c), where D is

* At present, research fellow, California Institute of Technology.

[1] F. R. Hirsh, Jr., J. O. S. A. **25**, 229 (1935).

* These ultraviolet data are from plate calibration curves taken by Dr. G. B. Sabine (now of the Eastman Kodak Research Laboratories) and very kindly given to the author.

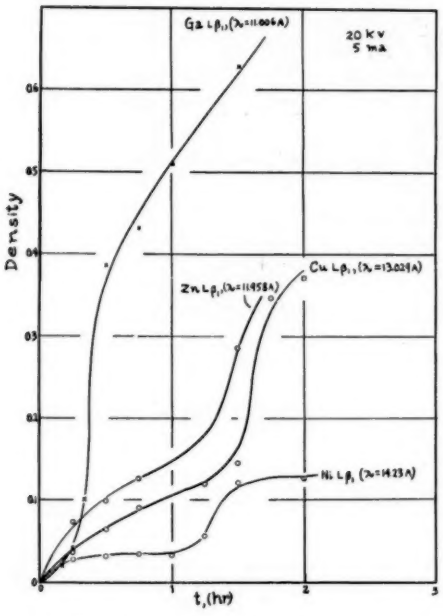

FIG. 1 (a). Photographic density, D, plotted against exposure time, t, for very "soft" x-rays ($\lambda \sim 11$–14.0A).

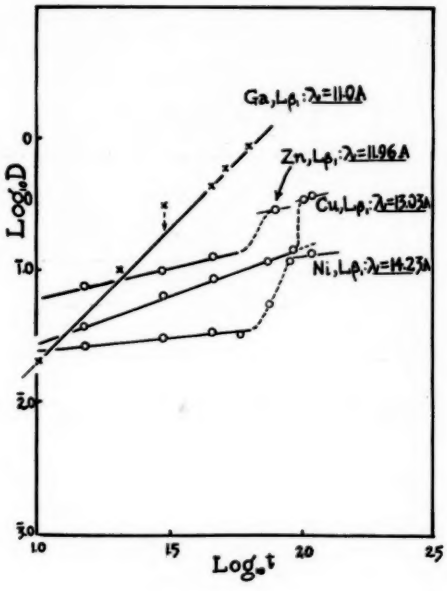

FIG. 1 (b). $\log_{10}$ density plotted against $\log_{10}$ exposure time using the data of Fig. 1 (a).

plotted against t and against $\log_{10} t$; and $\log_{10} D$ against $\log_{10} t$. The curves of D *vs.* t in Fig. 3(a) are similar to the familiar x-ray curves; the curves of Fig. 3(b) seem to be near the foot of the familiar "H & D" curves; Fig. 3(c) seems to suggest a new relation: $\log_{10} D$ *vs.* $\log_{10} t$ is linear.

The general equation of the straight lines in Fig. 1(b), Fig. 2 and Fig. 3(c) is:

$$\log_{10} D = p(\log_{10} t) + C, \qquad (1)$$

where C is a constant and p is the slope of the straight line. Taking antilogarithms of both sides we have

$$D = k_1 t^p,$$

where $C = \log_{10} k_1$. Let $k_1 = k_2 I_c$ where I_c is the constant x-ray intensity. Then

$$D = k_2 I_c t^p. \qquad (2)$$

In some of the curves plotted to give the linear relation (1), $p \sim 1$. If $p = 1$, Eq. (2) becomes

$$D = k_2(I_c t)$$

that of a straight line, and the D *vs.* t curve should have a linear region for "normal" exposure since then the factor in the brackets is the "exposure" and $D \propto$ exposure; this proportionality appears to be substantiated experimentally.[1] If $p > 1$ we have a density-exposure time

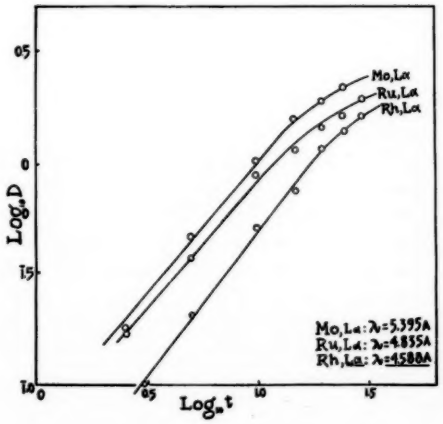

FIG. 2. The data of reference 1 replotted as $\log_{10}$ density *vs.* $\log_{10}$ exposure time, showing linear relation.

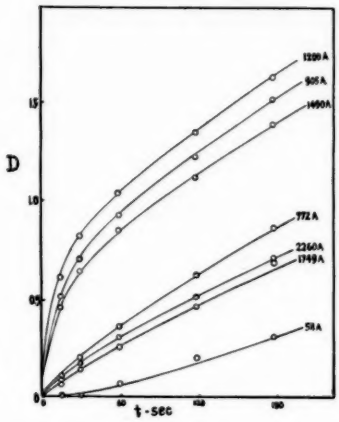

FIG. 3 (a). Density plotted against exposure time for certain ultraviolet wave-lengths, as designated.

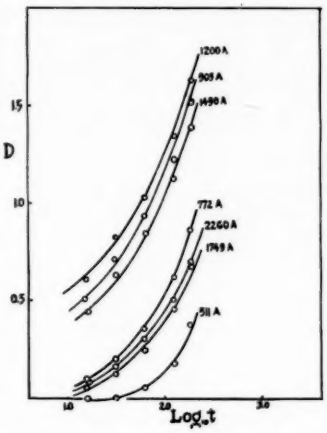

FIG. 3 (b). Density plotted against $\log_{10}$ exposure time, the familiar method of representation; for the same ultraviolet wave-lengths. Intensity is constant.

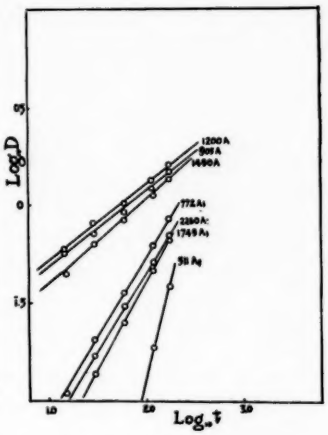

FIG. 3 (c). Log_{10} density plotted against $\log_{10}$ exposure time, showing the new linear relation.

curve which is concave upwards (see Figs. 3(a) and 3(c)); if $p<1$ the curve is concave downwards. For x-rays of moderate wave-length ($\sim$1–2A) it seems that $p\sim1$ and a reasonable assumption in this case is that $D\propto$ exposure. The magnitude of p probably depends on the method of development, and perhaps on the emulsion.

It is hoped that the linear relation may prove a useful tool in photographic photometry.

The author wishes to express his appreciation of the kindness of Dean F. K. Richtmyer in permitting the author to use certain experimental equipment granted Dean Richtmyer by the Heckscher Research Council of Cornell University.

Subtractive Color Mixture and Color Reproduction*

DAVID L. MACADAM
Eastman Kodak Research Laboratories, Rochester, New York
(Received August 24, 1938)

The colors of mixtures of certain sets of dyes can be predicted to a close approximation through the use of a simple rule. The analytical form of this rule is sufficiently simple to permit its incorporation into the theory of color reproduction. This inclusion of the phenomena of subtractive mixture in the theory leads directly to the conclusion that masking is an essential step in any realizable subtractive process of correct color reproduction. Formulas for the calculation of the gradients of the masks are given in terms of the parameters of the rule for the prediction of the colors of mixtures of the dyes to be used in printing the subtractive reproduction. Formulas are also given for the calculation of the gradients of masks to be used for the simultaneous introduction of partially negative spectral sensitivities. An application of these formulas to the Eastman wash-off relief process shows that the negative portions of the theoretically desirable spectral sensitivities are negligible, and that each separation negative need be printed in register with only one mask.

THE solution of many color problems would be greatly simplified if the colorimetric specifications of mixtures of dyes (and other colorants) could be computed from a limited number of parameters depending upon the colors of the components. Such a method of computation would be analogous to the method now available for the computation of the specifications of the color of additive mixtures from the colorimetric specifications of the additive components. The following discussion will show how the colors of subtractive mixtures of certain sets of dyes can be predicted with surprising accuracy. In this paper, this method of approximation will be applied to only one problem, namely, the improvement of subtractive color reproductions.[1] The application of this method to other color problems, such as the systematization of color matching, and the "strength testing" of new dyes and of new batches of familiar dyes, should not be difficult for those familiar with those problems and with the fundamentals of colorimetry.

The approximation method which will be presented is not rigorously exact in any known case, and is not generally applicable to all dyes. Consequently, this method cannot be regarded as a rigorous law of colorimetry. For brevity of expression, however, in the following discussion of a case in which it is usefully applicable, it will be referred to as the "law of subtractive color mixture." Obviously, in this and in every other application of this method, conclusions drawn from the use of the analytical expression of the law will be subject to limitations of accuracy. These limitations correspond to the limits of the range of accurate predictions of the colors of mixtures of the particular set of dyes with which each discussion is concerned.

Although the predictions are less accurate when the law is formulated and applied for wide variations of the concentrations of three component dyes, the accuracy attained in the cases of some suitably chosen sets of dyes is sufficient to render the law valuable for the improvement of three-color reproduction methods using these dyes. The law may prove applicable to some sets of transparent pigments, and should eliminate the necessity for hand correction of the plates used for three-color printing with such pigments. From this it can be concluded that the most satisfactory sets of practical dyes or pigments for use in subtractive color reproduction will be those which conform satisfactorily to the proposed law of subtractive color mixture, over the greatest ranges of desirable chromaticities and brightnesses.

A. Corrections for Base Color and for First Surface Reflections

Two prints, otherwise identical, one having a glossy finish and the other having a dull diffusing finish, are usually accepted as equivalent as far

* Communication No. 681 from the Kodak Research Laboratories.

[1] D. L. MacAdam, "Photographic Aspects of the Theory of Three-Color Reproduction," J. Opt. Soc. Am. **28**, 399 (1938).

as the color rendering is concerned, although the saturation, as well as the brightness contrast, is obviously degraded by the diffuse first surface reflectance of the dull print. This tendency to discount any observed first surface reflection of white light can be simulated if the spectral reflectance values of the darkest black print obtainable with any base are subtracted from the corresponding reflectance readings for all other colors printed on that base. The spectral reflectance of the unprinted base should be similarly decreased before it is used as the reflectance standard for the elimination of base color, as described in the following paragraph.

After the effects of first surface reflections have been discounted, the colors of a print of transparent colorants on a slightly yellow paper base are the same as the colors of a print similarly prepared on a perfectly colorless base, if the first print is observed in an illumination having a slightly different energy distribution, of higher color temperature than that used in observing the print on the colorless base. The effects of the color of the base can be eliminated if all spectral reflectances are measured relative to the reflectance of the unprinted base. The results will be equivalent to observing the print under an illuminant having a slightly higher color temperature than that assumed in the calculations. This slight increase in effective color temperature is negligible because of the well-known constancy of color relationships with moderate changes of illuminant quality.

The corrections for base color and for the elimination of first surface reflections are necessary before the proposed law of subtractive color mixture can be applied to the colors of dyes and dye mixtures determined from prints on any opaque support.

B. Law of Subtractive Color Mixture

The law of subtractive color mixture will be formulated for the case of mixtures of the three dye colors, yellow (Y), magenta (M), and blue-green (cyan, C), commonly used in three-color reproduction. The law can be applied more successfully to some sets of such dyes than to others. Since the law leads to a simple and successful method of automatic color correction, sets of dyes which obey the law to a close approximation will be more satisfactory for this type of color reproduction than sets of dyes whose mixtures are not satisfactorily predicted by any law of this form.

The daylight quality Illuminant C recommended by the International Commission on Illumination (abbreviated I.C.I.) will be assumed for all of the calculations and discussions in this paper. Equivalent results would follow from the use of any other practical illuminant.

Let R, G, and B, be the intensities of three suitably chosen primaries which, when additively mixed, match the color of the subtractive mixture of the dyes under consideration. Let these intensities, R, G, and B, be measured in such units that $R=G=B=1.0$, when the concentrations of all three dyes are zero, that is for "white." Then the *color density*, D_c, of any concentration of the cyan dye can be defined as the negative logarithm of the intensity, R_c, of the red primary in the additive mixture necessary to match the color of a print consisting of that concentration of the cyan dye printed alone. When the chromaticity, and units for measuring the intensity, R, are properly chosen, the color density, D_c, is almost exactly proportional to the concentration of the cyan dye. The color density of any concentration of the magenta, or of the yellow dye, is defined similarly, in terms of the primary predominantly controlled by each dye, thus:

$$D_c = -\log R_c, \tag{1.11}$$

$$D_m = -\log G_m, \tag{1.21}$$

$$D_y = -\log B_y. \tag{1.31}$$

The negative logarithms of the intensities of the other primaries necessary to match these separate prints of the dyes are found to be almost exactly proportional to the color densities. The six proportionality coefficients are defined by the expressions:

$$V_r{}^m D_m = -\log R_m, \tag{1.12}$$

$$V_r{}^y D_y = -\log R_y, \tag{1.13}$$

$$V_g{}^c D_c = -\log G_c, \tag{1.22}$$

$$V_g{}^y D_y = -\log G_y, \tag{1.23}$$

$$V_b{}^c D_c = -\log B_c, \tag{1.32}$$

$$V_b{}^m D_m = -\log B_m. \tag{1.33}$$

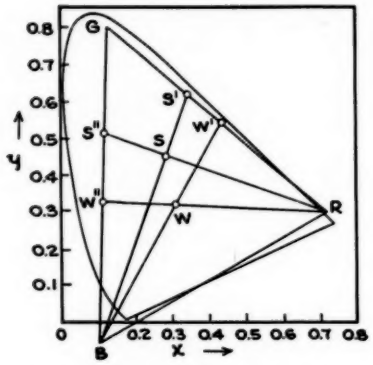

FIG. 1. Chromaticity diagram illustrating geometrical relations on which the logarithmic coordinate system is based.

The quantities, R_c, G_c, and B_c are the intensities of the red, green, and blue primaries necessary to match the color of the cyan dye printed alone. Similarly, R_m, G_m, and B_m, are the intensities of the red, green, and blue primaries necessary to match the color of the magenta component printed alone. Finally, R_y, G_y, and B_y are the intensities of the primaries necessary to match the color of the yellow component printed alone. If the concentration of each dye in the separate prints is the same as the concentration of that dye in a combined print of all three dyes, then the negative logarithms of the intensities of the primaries, R, G, and B, necessary to match the color of this subtractive mixture will be given almost exactly by the following linear combinations of the color densities of the component dyes:

$$-\log R = D_c + V_r^m D_m + V_r^y D_y, \qquad (1.1)$$

$$-\log G = V_g^c D_c + D_m + V_g^y D_y, \qquad (1.2)$$

$$-\log B = V_b^c D_c + V_b^m D_m + D_y. \qquad (1.3)$$

Equations (1.1), (1.2), and (1.3) are the first form of the proposed law for the approximate prediction of the colors of subtractive mixtures. The problem of choosing the chromaticities of the primaries, R, G, and B will be discussed later. The intensities, R, G, and B, of these primaries are to be computed from the I.C.I. tristimulus values for each color. This can only be done after the chromaticities of R, G, and B are known in terms of the I.C.I. primaries. The chromaticities which should be chosen for the primaries, R, G, and B, in order to render the predictions of Eqs. (1.1), (1.2), and (1.3) most consistent with the observed colors of mixtures of the dyes will depend upon the spectrophotometric characteristics of the dyes as well as upon the characteristics of human vision. The primaries, R, G, and B are not necessarily within the gamut of physically attainable colors. The values of the coefficients, V_r^m, V_r^y, V_g^c, V_g^y, V_b^c, and V_b^m, defined by Eqs. (1.12), (1.13), (1.22), (1.23),

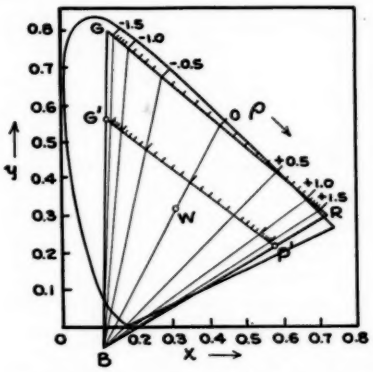

FIG. 2a. Chromaticity diagram illustrating the projectivity of the logarithmic coordinate scale, ρ.

(1.32), and (1.33) depend upon the choice of the primaries, R, G, and B, as well as upon the spectrophotometric characteristics of the dyes.

The chromaticity of any color can be specified by the two ratios of intensities, R/G, and B/G, or by any single valued functions of these ratios. The second form of the proposed law of subtractive mixtures is most convenient if the chromaticities are specified in terms of the two logarithmic parameters,*[2] thus:

* Sinden proposed the use of similar functions of the intensities of specially selected primaries. His suggestion was based on an extension of the Weber-Fechner law of just noticeable intensity differences, as if it were applicable to the Young-Helmholtz theory of color vision. Sinden's purpose was to secure a more accurate method of representing just noticeable color differences. Our present purpose is to simplify the prediction of the chromaticities of subtractive color mixtures. The analytical similarity of the parameters used by Sinden to those in the present discussion is no more than a formal similarity. The primaries suitable for Sinden's purpose must be uniquely determined from studies of the chromaticity sensibility of human vision. The primaries suitable for our present purpose will be characteristic of the set of dyes being studied, and other sets of dyes will require other sets of primaries. See reference 2.

[2] R. H. Sinden, "Further Search for the Ideal Color System, I," J. Opt. Soc. Am. **27**, 124 (1937).

$$\rho = \log (R/G), \tag{2.1}$$

$$\beta = \log (B/G). \tag{2.2}$$

Let ρ_c, β_c, ρ_m, β_m, ρ_y, β_y, ρ, and β denote the values of the parameters defined by Eqs. (2.1) and (2.2) for the separate prints of the component dyes of the mixture, and for the composite print of the mixture. Since $\rho = \log R - \log G$, and $\beta = \log B - \log G$, substitution of Eqs. (1.1), (1.2), and (1.3) leads to the following values of ρ and β:

$$\rho_c = (V_g{}^c - 1)D_c, \tag{2.11}$$

$$\beta_c = (V_g{}^c - V_b{}^c)D_c, \tag{2.21}$$

$$\rho_m = (1 - V_r{}^m)D_m, \tag{2.12}$$

$$\beta_m = (1 - V_b{}^m)D_m, \tag{2.22}$$

$$\rho_y = (V_g{}^y - V_r{}^y)D_y, \tag{2.13}$$

$$\beta_y = (V_g{}^y - 1)D_y, \tag{2.23}$$

$$\rho = \rho_c + \rho_m + \rho_y, \tag{2.14}$$

$$\beta = \beta_c + \beta_m + \beta_y. \tag{2.24}$$

Equations (2.14) and (2.24) are the simplest possible statement of the proposed law of subtractive color mixture. The parameters ρ_c and β_c are proportional to the color density of the cyan dye printed alone, and consequently are nearly proportional to the concentration of the cyan dye in the composite image. Similarly, ρ_m and β_m are proportional to D_m, and almost proportional to the concentration of the magenta dye in the image. Finally, ρ_y and β_y are proportional to D_y, and almost proportional to the concentration of the yellow dye.

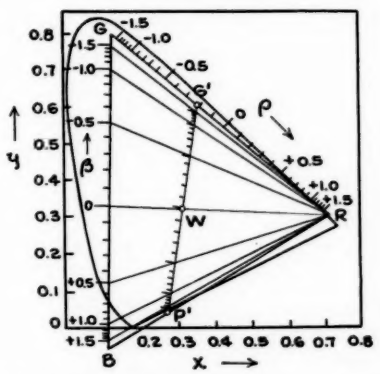

FIG. 2b. Chromaticity diagram illustrating the projectivity of the logarithmic coordinate scale, β.

It is not necessary to determine the absolute values of D_c, D_m, and D_y or of the V's, if only the chromaticity of the mixture is desired. This follows from the easily proved rule that the parameters ρ and β for any color represented by the point, S, in the chromaticity diagram, such as shown in Fig. 1, are the logarithms of anharmonic ratios of segments of the lines GR and GB, respectively:

$$\rho = \log \left(\frac{GS'}{S'R} \cdot \frac{RW'}{W'G} \right), \tag{2.15}$$

$$\beta = \log \left(\frac{GS''}{S''B} \cdot \frac{BW''}{W''G} \right). \tag{2.25}$$

The points, R, G, and B, represent the chromaticities of the primaries which permit the most accurate predictions of the colors of mixtures of the dyes, according to Eqs. (1.1), (1.2), (1.3), (2.14), and (2.24). The point, W, represents the chromaticity of the support, for which $\rho = \beta = 0$. The points, S' and W', are the intersections of the line GR, with the lines drawn from the point, B, through the points, S and W. Similarly, the points, S'' and W'', are the intersections of the line, GB, with the line drawn from the point, R, through the points, S and W. The parameters, ρ and β, which permit the predictions of the colors of subtractive mixtures of certain sets of dyes can therefore be determined graphically from the locations of the points R, G, B, W, and S, in the I.C.I. chromaticity diagram. Since S is the only variable point when the discussion is confined to one set of dyes, scales of the param-

TABLE I. *Master scale of ρ and β.*

ρ OR β	$S'''P'$	ρ OR β	$S'''P'$	ρ OR β	$S'''P'$
0.00	0.500	0.55	0.220	1.05	0.082
0.05	0.471	0.60	0.201	1.10	0.074
0.10	0.442	0.65	0.183	1.15	0.066
0.15	0.414	0.70	0.166	1.20	0.059
0.20	0.387	0.75	0.151	1.25	0.053
0.25	0.360	0.80	0.137	1.30	0.048
0.30	0.334	0.85	0.124	1.35	0.043
0.35	0.309	0.90	0.112	1.40	0.038
0.40	0.285	0.95	0.101	1.45	0.034
0.45	0.262	1.00	0.091	1.50	0.031
0.50	0.240			2.00	0.010

eters, ρ and β, can be constructed along the lines, GR and GB. These scales are most easily constructed by projecting the scale $G'P'$ onto GR and GB, using B and R, respectively, as the centers of projection, as shown in Figs. 2(a) and 2(b). The scale of ρ and β on $G'P'$ is given by $\rho=\beta=\log(G'S'''/S'''P')$. The projectivity of this scale, as employed in Figs. 2(a) and 2(b), follows from the theorems of projective geometry. Table I shows the values of $S'''P'$ for positive values of ρ and β, when $G'P'=1.000$. The distance, $G'S'''$, takes these same values for the corresponding negative values of ρ and β. The coordinate grid for determining ρ and β for any given set of dyes can be drawn permanently in the standard I.C.I. coordinate system. This coordinate grid can be used for all predictions of the colors of subtractive mixtures of these dyes. The values of ρ and β can be determined in a moment from the known chromaticities or concentrations of the component dyes for which the coordinate grid has been constructed. The predicted chromaticity of the mixture is the point having ρ and β equal to the sum of the values of ρ and β for the components.

C. Determination of Primaries for Use with the Law of Subtractive Mixtures

The following procedure is recommended for the determination of the primaries, R, G, and B, for any practical set of colorants. The spectral reflectances of separate prints of each of three dyes[3] were measured relative to the spectral

[3] Eastman Kodak Company, "Color Printing with the Eastman Wash-Off Relief Process," Rochester, 1938.

TABLE II. *Colorimetric specifications and predictions of three-color subtractive mixtures.*

	Observed Values							Predicted Values			
Composition	X	Y	Z	x	y	ρ	β	ρ	β	x	y
C	0.6937	0.7341	1.0641	0.2780	0.2947	−0.07	+0.06				
$2C$	0.5026	0.5479	0.9600	0.2493	0.2725	−0.14	+0.12	−0.14	+0.12	0.249	0.273
$4C$	0.2845	0.3171	0.7803	0.2058	0.2290	−0.28	+0.24	−0.28	+0.24	0.206	0.229
$8C$	0.1220	0.1251	0.5433	0.1545	0.1586	−0.56	+0.38	−0.56	+0.48	0.150	0.142
M	0.8455	0.7747	1.0315	0.3185	0.2923	+0.08	+0.08				
$2M$	0.7395	0.6087	0.9020	0.3286	0.2705	+0.16	+0.15	+0.16	+0.15	0.329	0.271
$4M$	0.5860	0.3959	0.7033	0.3473	0.2345	+0.32	+0.31	+0.32	+0.30	0.343	0.233
$8M$	0.4125	0.2063	0.4468	0.3870	0.1937	+0.64	+0.58	+0.64	+0.60	0.382	0.187
Y	0.9340	0.9841	0.9245	0.3280	0.3460	+0.00	−0.10				
$2Y$	0.8985	0.9705	0.7223	0.3470	0.3750	+0.01	−0.20	+0.01	−0.20	0.347	0.375
$4Y$	0.8500	0.9492	0.4415	0.3783	0.4233	+0.02	−0.40	+0.02	−0.40	0.378	0.423
$8Y$	0.8060	0.9197	0.1919	0.4193	0.4793	+0.04	−0.80	+0.04	−0.80	0.419	0.479
$C+M$	0.5957	0.5651	0.9265	0.2850	0.2700	0.00	+0.14	+0.01	+0.14	0.289	0.273
$2C+2M$	0.3703	0.3215	0.7300	0.2603	0.2260	0.00	+0.28	+0.02	+0.27	0.266	0.230
$4C+4M$	0.1592	0.1085	0.4564	0.2202	0.1498	0.00	+0.53	+0.04	+0.54	0.219	0.146
$8C+8M$	0.0423	0.0156	0.1850	0.1741	0.0643	0.00	+0.90	+0.08	+1.08	0.165	0.046
$C+Y$	0.6520	0.7213	0.8290	0.2957	0.3280	−0.07	−0.04	−0.07	−0.04	0.296	0.328
$2C+2Y$	0.4358	0.5261	0.5838	0.2820	0.3405	−0.13	−0.08	−0.13	−0.08	0.282	0.341
$4C+4Y$	0.1982	0.2842	0.2954	0.2550	0.3645	−0.26	−0.13	−0.26	−0.16	0.262	0.376
$8C+8Y$	0.0444	0.0918	0.0811	0.2042	0.4223	−0.56	−0.25	−0.52	−0.32	0.240	0.446
$M+Y$	0.8050	0.7628	0.7978	0.3403	0.3225	+0.08	−0.03	+0.08	−0.02	0.342	0.323
$2M+2Y$	0.6760	0.5923	0.5400	0.3740	0.3275	+0.17	−0.06	+0.17	−0.05	0.377	0.329
$4M+4Y$	0.5045	0.3775	0.2480	0.4462	0.3340	+0.34	−0.12	+0.34	−0.10	0.444	0.334
$8M+8Y$	0.3400	0.1970	0.0536	0.5763	0.3335	+0.70	−0.24	+0.68	−0.20	0.570	0.331
$M+0.8Y$	0.8133	0.7659	0.8410	0.3360	0.3166	+0.08	−0.01	+0.08	−0.01	0.340	0.316
$2M+1.6Y$	0.6855	0.5955	0.5990	0.3642	0.3165	+0.16	−0.02	+0.17	−0.01	0.370	0.316
$4M+3.2Y$	0.5175	0.3822	0.3050	0.4290	0.3170	+0.34	−0.04	+0.34	−0.02	0.428	0.315
$8M+6.4Y$	0.3450	0.1984	0.0813	0.5528	0.3177	+0.69	−0.08	+0.68	−0.04	0.540	0.312
$C+0.6Y$	0.6655	0.7254	0.9165	0.2890	0.3150	−0.07	0.00	−0.07	0.00	0.294	0.317
$2C+1.2Y$	0.4575	0.5329	0.7118	0.2685	0.3130	−0.13	+0.01	−0.14	+0.01	0.272	0.315
$4C+2.4Y$	0.2222	0.2955	0.4340	0.2350	0.3115	−0.27	+0.01	−0.27	+0.01	0.240	0.314
$8C+4.8Y$	0.0588	0.1003	0.1640	0.1818	0.3110	−0.58	+0.02	−0.54	+0.02	0.188	0.312

reflectance of the undyed paper support, and these reflectances were corrected for the elimination of first surface reflection, as discussed in Section A. The colorimetric specifications computed for the colors, C, Y, and M, represented in Figs. 3, 4, and 5 are given in Table II. The colorimetric specifications of each of these dyes printed with doubled concentration are given in Table II, following the designations $2C$, $2M$, and $2Y$. The colorimetric specifications of the colors resulting from four and eight times the original concentrations are also given in Table II. These colors are represented on the chromaticity diagram shown in Fig. 6. All three loci are slightly curved. The chromaticities of mixtures of the cyan and yellow dyes, $C+Y$, $2C+2Y$, $4C+4Y$, and $8C+8Y$, were computed, and are recorded in Table II and represented in Fig. 6. Similarly, the colorimetric specifications of 1 : 1 mixtures of cyan and magenta, and of magenta and yellow were computed, and included in Table II and Fig. 6. The locus of chromaticities of mixtures of equal concentrations of cyan and magenta is almost a straight line. The loci of the other two 1 : 1 mixtures are only slightly less straight. It was estimated from the trends of the curvatures that the loci of the mixtures: $n(M+0.8Y)$, $n(C+0.6Y)$, $n(C+2.0Y)$, and $n(M+0.1C)$, should be almost straight lines. These chromaticities were computed and are given in Table II and Fig. 6. Although none are exactly straight lines, only the locus designated $n(C+2.0Y)$ was sufficiently curved to require a closer approximation. The indicated chromaticities on the locus denoted $n(C+1.5Y)$ were first computed, and finally those along the locus $n(C+1.4Y)$ were computed and accepted as sufficiently close to a straight line. According to

TABLE II.—*Continued*

Composition	Observed Values							Predicted Values			
	X	Y	Z	x	y	ρ	β	ρ	β	x	y
$4M+0.4C$	0.5045	0.3424	0.6700	0.3325	0.2258	+0.29	+0.33	+0.29	+0.33	0.333	0.223
$8M+0.8C$	0.2993	0.1484	0.4023	0.3523	0.1737	+0.58	+0.64	+0.58	+0.66	0.353	0.169
$C+2Y$	0.6200	0.7102	0.6505	0.3130	0.3590	−0.06	−0.14	−0.06	−0.14	0.318	0.360
$2C+4Y$	0.3960	0.5076	0.3597	0.3133	0.4095	−0.13	−0.29	−0.12	−0.28	0.318	0.405
$4C+8Y$	0.1695	0.2674	0.1223	0.3040	0.4790	−0.25	−0.53	−0.24	−0.56	0.313	0.488
$8C+16Y$	0.0355	0.0810	0.0204	0.2595	0.5925	−0.50	−0.88	−0.48	−1.12	0.272	0.620
$C+1.5Y$	0.6340	0.7136	0.7325	0.3045	0.3430	−0.06	−0.08	−0.06	−0.09	0.308	0.345
$2C+3Y$	0.4140	0.5160	0.4613	0.2970	0.3700	−0.13	−0.16	−0.13	−0.18	0.302	0.376
$4C+6Y$	0.1802	0.2745	0.1883	0.2805	0.4270	−0.26	−0.31	−0.25	−0.36	0.288	0.433
$8C+12Y$	0.0384	0.0856	0.0366	0.2390	0.5340	−0.52	−0.60	−0.50	−0.72	0.251	0.560
$C+1.4Y$	0.6375	0.7141	0.7555	0.3025	0.3395	−0.06	−0.07	−0.06	−0.08	0.305	0.342
$2C+2.8Y$	0.4168	0.5160	0.4833	0.2942	0.3643	−0.13	−0.14	−0.13	−0.16	0.299	0.372
$4C+5.6Y$	0.1830	0.2761	0.2050	0.2757	0.4160	−0.26	−0.28	−0.26	−0.32	0.283	0.477
$8C+11.2Y$	0.0394	0.087	0.0430	0.2327	0.5140	−0.54	−0.53	−0.52	−0.64	0.244	0.545
$8C+C+1.4Y$	0.0798	0.0957	0.3385	0.1550	0.1862	−0.60	+0.33	−0.63	+0.40	0.150	0.164
$8C+2C+2.8Y$	0.0521	0.0708	0.2195	0.1520	0.2068	−0.66	+0.28	−0.69	+0.32	0.151	0.193
$8C+4C+5.6Y$	0.0231	0.0403	0.0943	0.1463	0.2555	−0.85	+0.16	−0.82	+0.16	0.150	0.255
$8C+8C+11.2Y$	0.0048	0.0141	0.0191	0.1261	0.3710	−1.50	−0.10	−1.08	−0.16	0.146	0.397
$8Y+C+1.4Y$	0.5300	0.6563	0.1350	0.4010	0.4975	−0.02	−0.85	−0.03	−0.88	0.403	0.500
$8Y+2C+2.8Y$	0.3500	0.4726	0.0941	0.3820	0.5150	−0.08	−0.90	−0.09	−0.96	0.385	0.523
$8Y+4C+5.6Y$	0.1570	0.2509	0.0440	0.3480	0.5560	−0.21	−1.05	−0.22	−1.12	0.350	0.565
$8Y+8C+11.2Y$	0.0348	0.0788	0.0141	0.2730	0.6170	−0.48	−1.12	−0.48	−1.44	0.278	0.645
$8M+M+0.8Y$	0.3688	0.1758	0.3215	0.4260	0.2030	+0.77	+0.57	+0.73	+0.60	0.419	0.200
$8M+2M+1.6Y$	0.3340	0.1590	0.2320	0.4610	0.2192	+0.84	+0.56	+0.81	+0.59	0.457	0.217
$8M+4M+3.2Y$	0.2883	0.1332	0.1234	0.5295	0.2445	+1.02	+0.54	+0.98	+0.58	0.515	0.235
$8M+8M+6.4Y$	0.2293	0.1013	0.0352	0.6255	0.2757	+1.42	+0.50	+1.32	+0.56	0.615	0.272
$8Y+M+0.8Y$	0.6885	0.7079	0.1366	0.4495	0.4625	+0.13	−0.81	+0.13	−0.81	0.450	0.462
$8Y+2M+1.6Y$	0.5958	0.5554	0.0958	0.4770	0.4443	+0.21	−0.81	+0.21	−0.81	0.477	0.444
$8Y+4M+3.2Y$	0.4713	0.3627	0.0476	0.5342	0.4115	+0.39	−0.81	+0.38	−0.82	0.534	0.411
$8Y+8M+6.4Y$	0.3318	0.1877	0.0112	0.6260	0.3540	+0.79	−0.81	+0.72	−0.84	0.615	0.360

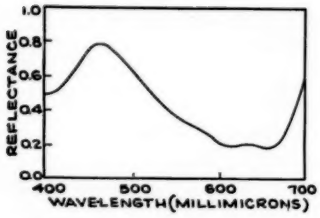

FIG. 3. Spectral reflectance curve of a typical print of a cyan (minus red) image component.

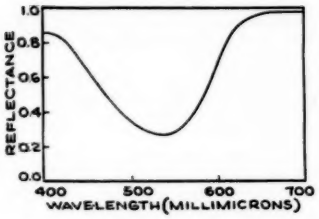

FIG. 4. Spectral reflectance curve of a typical print of a magenta (minus green) image component.

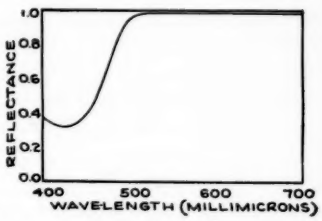

FIG. 5. Spectral reflectance curve of a typical print of a yellow (minus blue) image component.

the proposed law of subtractive mixtures, ρ should be proportional to β along every locus of mixtures whose percentage compositions remain constant while the total concentration varies. An examination of the properties of the ρ, β coordinate system reveals that loci along which ρ is proportional to β can be straight lines only when $\rho=0$, or when $\beta=0$, or when $\rho=\beta$, and that such loci should be coincident with the straight lines joining the white point, W, with each of the primaries B, R, and G.

The chromaticities of the mixtures of $8Y$ with each of the mixtures on the straight locus designated $n(C+1.4Y)$ were computed, and are designated in Table II and Fig. 6 by $8Y+n(C+1.4Y)$, where n takes the values 1, 2, 4, and 8. Similarly, the mixtures of $8C$ with each of the mixtures on the straight locus designated $n(C+1.4Y)$ were computed, and are designated in Table II and Fig. 6 by $8C+n(C+1.4Y)$. A continued examination of the properties of the ρ, β coordinate system will reveal that the proposed law of subtractive mixtures predicts that these two new loci will be straight lines, having a common intersection with the original straight locus, $n(C+1.4Y)$. These properties should be possessed by every locus, $pY+n(C+1.4Y)$, or $qC+n(C+1.4Y)$, for which p or q can have any value which is fixed, while n varies during the tracing of the locus. The common intersection of all of these straight loci should be the primary, G, which is most suitable for use with the subtractive law. From the three loci which have been computed, which possess the properties predicted by the subtractive law, the green primary, G, for the dyes specified in Figs. 3, 4, and 5, is found to be at the point, $x=0.125$, $y=0.800$, in the I.C.I. chromaticity diagram.

Since the locus, nY, of the chromaticities of various concentrations of the yellow dye alone was nearly straight, loci of chromaticities of mixtures having the compositions, $pC+nY$, and $qM+nY$, where n alone varies for different points on each locus, should therefore be nearly straight. Several of such loci were traced through points which have already been mentioned and specified in Table II. These loci are shown in Fig. 6, where they are seen to intersect in a single point which is also on the straight locus, $n(C+M)$. According to the subtractive law, this point of intersection, which is at $x=0.113$, $y=-0.045$, is the blue primary, B. It will be noticed that this primary is outside the gamut of physically attainable colors. This property of the blue primary does not diminish the usefulness of the subtractive law, since there will never be any necessity to produce the primary for physical experiments. On the contrary, the super-spectrum saturation of the hypothetical blue primary will be of advantage in minimizing the negative portions of the theoretically desirable spectral sensitivities[1, 4] for the photographic process which will utilize the effective primaries, R, G, and B.

The mixture of the colors of the locus, $n(M+0.8C)$, first with $8M$ and then with $8Y$ also results in straight chromaticity loci, intersecting in the single point, $x=0.710$, $y=0.300$, which is therefore taken as the red primary, R, for use with the subtractive law. As in the case of the blue primary, the red primary lies outside the gamut of physically attainable colors.

The coordinate grid for the rapid determination of ρ and β for these dyes can now be con-

[4] A. C. Hardy and L. Wurzburg, "The Theory of Three-Color Reproduction," J. Opt. Soc. Am. 27, 227 (1937).

structed. The values of ρ and β for the various mixtures whose chromaticities are represented in Fig. 6 are recorded in Table II. The values of ρ and β predicted from the values of ρ and β for the components are also recorded, and the predicted values of the coordinates, x and y, of these mixtures are recorded for comparison with the observed values. Discrepancies are evident in the table, but the over-all accuracy of prediction is quite encouraging and useful. The customary errors of subtractive color reproduction are from five to ten times as great as the errors shown in Table II, and a successful incorporation of the proposed law of subtractive color mixture into the theory and practice of color photography should result in marked improvements in color reproduction.

The intensities, R, G, and B, of the primaries necessary to match any color can be computed from the I.C.I. tristimulus values by use of the following equations:

$$R=1.46X-0.21Y-0.187\ Z, \qquad (3.1)$$

$$G=-0.60X+1.42Y+0.14\ Z, \qquad (3.2)$$

$$B=0.05X-0.09Y+0.88\ Z. \qquad (3.3)$$

These equations were derived in the customary manner from the chromaticities of the primaries, R, G, and B, and from the requirement that $R=G=B=1.0$, in order to match the color of the undyed support. The illumination is assumed to be suitably modified from the standard Illuminant C in order to compensate for the absorption of the support. Therefore, the I.C.I. tristimulus values of the support are: $X=0.9804$, $Y=1.000$, and $Z=1.1812$, matching the chromaticity of the standard Illuminant C. This compensation is most conveniently introduced into the measurements and calculations when the undyed support is used as the standard (100 percent) of spectral reflectance (or transmittance).

The values of R, G, and B computed from the values of X, Y, and Z given in Table II for the various concentrations of the separate dyes, C, M, and Y, are shown in Table III. The negative logarithms of the R, G, and B values are also listed. It will be seen that these "densities" are nearly proportional to the concentration, as required for the successful application of the proposed law of subtractive mixtures.

When the proper values from Table III are substituted in Eqs. (1.11), (1.12), (1.13), (1.21), (1.22), (1.23), (1.31), (1.32), and (1.33), the four sets of values of the proportionality coefficients shown in Table IV are obtained.

Since these values are not completely independent of concentration, some criterion must be adopted for the selection of a value for each of these coefficients for use in the analysis which is to follow. The values adopted and given in Table V agree with the observed values for the lowest concentrations, and are smoothed to the second decimal place because no higher precision is justified by the data.

D. Application of the Law of Subtractive Color Mixture to the Theoretical Requirements for Subtractive Color Reproduction

Let us consider the reproduction secured with a perfect additive process employing the hypothetical projection primaries, R, G, and B, which were discussed in the previous sections. The

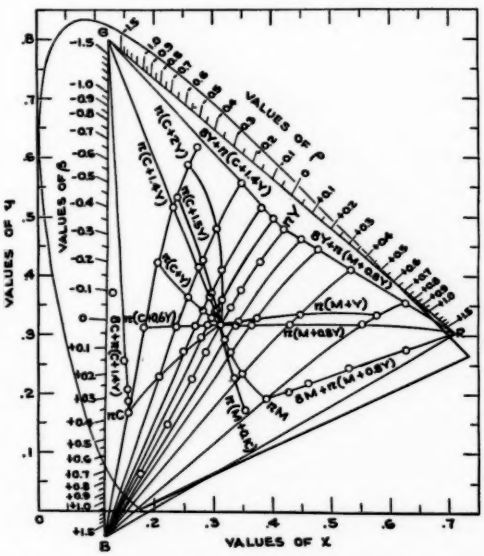

FIG. 6. Chromaticity diagram, indicating the chromaticities of various mixtures of cyan, magenta, and yellow image components. The constructions used for the determination of the corresponding subtractive primaries, R, G, and B, are shown.

amounts, R, G, and B, of each of these primaries entering into the synthesis of every correctly reproduced color will be[1, 4]

$$R=\int F_\lambda E_\lambda \bar{r}_\lambda d\lambda, \tag{4.1}$$

$$G=\int F_\lambda E_\lambda \bar{g}_\lambda d\lambda, \tag{4.2}$$

$$B=\int F_\lambda E_\lambda \bar{b}_\lambda d\lambda. \tag{4.3}$$

In these expressions, the spectral distribution of the energy reflected from the object whose color is being correctly reproduced is denoted by the product, $F_\lambda E_\lambda$, and the tristimulus values of the spectrum transformed to the projection primaries are represented by $\bar{r}_\lambda$, $\bar{g}_\lambda$, and $\bar{b}_\lambda$. The transformations result from the substitution of the I.C.I. functions, $\bar{x}_\lambda$, $\bar{y}_\lambda$, $\bar{z}_\lambda$, in Eqs. (3.1), (3.2), and (3.3). The effective spectral sensitivities of the process used in preparing the images for projection must have been proportional to $\bar{r}_\lambda$, $\bar{g}_\lambda$, and $\bar{b}_\lambda$. If the reproduction has been secured by the conventional negative-to-positive photographic process, and if all of the densities are on the straight-line portion of the H and D curves, then the densities of the three negatives must be linear functions of log R, log G, and log B, respectively.

As explained in an earlier paper,[1] it is convenient to convert the exposures, R, G, and B, to inertia units. This conversion is accomplished by multiplying the values of R, G, and B by some single factor whose magnitude depends upon the exposure latitude of the photographic process. If the logarithm of this factor is denoted by D_o, and if the exposures expressed in inertia units are represented by the bold-faced capitals, **R**, **G**, and **B**, then

$$\log \mathbf{R}=D_o+\log R, \tag{4.11}$$

$$\log \mathbf{G}=D_o+\log G, \tag{4.21}$$

$$\log \mathbf{B}=D_o+\log B. \tag{4.31}$$

The densities of the three separation negatives will then be

$$D_{rn}=\gamma_{rn}\log \mathbf{R}, \tag{4.12}$$

$$D_{gn}=\gamma_{gn}\log \mathbf{G}, \tag{4.22}$$

$$D_{bn}=\gamma_{bn}\log \mathbf{B}. \tag{4.32}$$

The densities of positive transparencies prepared from these negatives will be:

$$D_{rp}=D_{op}-\gamma_{rp}\gamma_{rn}\log \mathbf{R}, \tag{4.13}$$

$$D_{gp}=D_{op}-\gamma_{gp}\gamma_{gn}\log \mathbf{G}, \tag{4.23}$$

$$D_{bp}=D_{op}-\gamma_{bp}\gamma_{bn}\log \mathbf{B}. \tag{4.33}$$

The density, D_{op}, is the density of the positive wherever the density of the negative is zero, and the γ's are the gradients of density-*versus*-log exposure to which the indicated negatives and positives are developed. In an additive reproduction, the products of the respective pairs of γ's must be equal to unity, and equal intensities of the primaries should be projected on the screen, resulting in neutral mixtures, whenever $D_{rp}=D_{gp}=D_{bp}$.

In order to render the subtractive process colorimetrically equivalent to the additive process, within the gamut of colors attainable with the dyes, it is only necessary to print suitable

TABLE III. *Color densities of subtractive image components.*

SAMPLE	R	$-\log R$	G	$-\log G$	B	$-\log B$
C	0.660	0.180	0.774	0.111	0.906	0.043
$2C$	0.439	0.357	0.611	0.214	0.821	0.086
$4C$	0.203	0.692	0.389	0.410	0.672	0.173
$8C$	0.050	1.301	0.181	0.742	0.472	0.326
M	0.879	0.056	0.737	0.132	0.881	0.055
$2M$	0.771	0.113	0.548	0.262	0.776	0.110
$4M$	0.641	0.193	0.310	0.508	0.611	0.214
$8M$	0.474	0.324	0.108	0.967	0.394	0.405
Y	0.985	0.006	0.966	0.015	0.771	0.113
$2Y$	0.971	0.012	0.942	0.026	0.593	0.227
$4Y$	0.958	0.019	0.902	0.045	0.346	0.460
$8Y$	0.945	0.025	0.861	0.065	0.126	0.900

TABLE IV. *Observed values of coefficients in law of subtractive mixture.*

FROM SAMPLE	$V_g{}^c$	$V_b{}^c$
C	0.615	0.238
$2C$	0.598	0.240
$4C$	0.592	0.250
$8C$	0.566	0.249
	$V_r{}^m$	$V_b{}^m$
M	0.424	0.416
$2M$	0.432	0.419
$4M$	0.380	0.422
$8M$	0.336	0.419
	$V_r{}^y$	$V_g{}^y$
Y	0.053	0.133
$2Y$	0.053	0.115
$4Y$	0.041	0.098
$8Y$	0.028	0.072

densities of the cyan, magenta, and yellow dyes such that the values for R, G, and B, which describe the color of the subtractive mixture, will be proportional to the exposures, **R**, **G**, and **B**, recorded by the ideal negatives. The constant, D_o, is the logarithm of the proportionality constant, having such a value that white objects are reproduced by undyed support in the subtractive reproduction. These conditions for correct color reproduction can be expressed analytically by equating the values of R, G, and B derived from Eqs. (1.1), (1.2), (1.3), (4.11), (4.21), (4.31), (4.12), (4.22), and (4.32). The resulting expressions are:

TABLE V. *Adopted values of coefficients in law of subtractive mixture.*

$V_g^c = 0.60$	$V_b^c = 0.24$
$V_r^m = 0.42$	$V_b^m = 0.42$
$V_r^y = 0.05$	$V_g^y = 0.12$

$$D_o - D_{rn}/\gamma_{rn} = D_c + V_r^m D_m + V_r^y D_y, \quad (5.1)$$

$$D_o - D_{gn}/\gamma_{gn} = D_m + V_g^c D_c + V_g^y D_y, \quad (5.2)$$

$$D_o - D_{bn}/\gamma_{bn} = D_y + V_b^c D_c + V_b^m D_m. \quad (5.3)$$

The necessary values of the color densities, D_c, D_m, and D_y can be found by solving Eqs. (5.1), (5.2), and (5.3) explicitly for these quantities:

$$D_c = D_o - (A/\gamma_{rn})(1 - V_b^m V_g^y)[D_{rn} - D_{gn}(\gamma_{rn}/\gamma_{gn})(V_r^m - V_b^m V_r^y)/(1 - V_b^m V_g^y) - D_{bn}(\gamma_{rn}/\gamma_{bn})(V_r^y - V_r^m V_g^y)/(1 - V_b^m V_g^y)], \quad (5.11)$$

$$D_m = D_o - (A/\gamma_{gn})(1 - V_b^c V_r^y)[D_{gn} - D_{rn}(\gamma_{gn}/\gamma_{rn})(V_g^c - V_b^c V_g^y)/(1 - V_b^c V_r^y) - D_{bn}(\gamma_{gn}/\gamma_{bn})(V_g^y - V_g^c V_r^y)/(1 - V_b^c V_r^y)], \quad (5.21)$$

$$D_y = D_o - (A/\gamma_{bn})(1 - V_r^m V_g^c)[D_{bn} - D_{rn}(\gamma_{bn}/\gamma_{rn})(V_b^c - V_b^m V_g^c)/(1 - V_r^m V_g^c) - D_{gn}(\gamma_{bn}/\gamma_{gn})(V_b^m - V_b^c V_r^m)/(1 - V_r^m V_g^c)]. \quad (5.31)$$

In these equations, $A = 1/[(1 - V_b^m V_g^y) - V_g^c(V_r^m - V_b^m V_r^y) - V_b^c(V_r^y - V_r^m V_g^y)]$. (5.41)

These equations are similar in form to Eqs. (4.13), (4.23), and (4.33) and can therefore be interpreted as representing positive images. The densities of these positive images are the desired color densities, D_c, D_m, and D_y. The densities of the *printing negatives*, D_{cn}, D_{mn}, and D_{yn}, are the three quantities enclosed in square brackets in Eqs. (5.11), (5.21), and (5.31), respectively. The printing gradients γ_c, γ_m, and γ_y, of the color densities with respect to the densities of the printing negatives should be:

$$\gamma_c = dD_c/dD_{cn} = (A/\gamma_{rn})(1 - V_b^m V_g^y), \quad (5.12)$$

$$\gamma_m = dD_m/dD_{mn} = (A/\gamma_{gn})(1 - V_b^c V_r^y), \quad (5.22)$$

$$\gamma_y = dD_y/dD_{yn} = (A/\gamma_{bn})(1 - V_r^m V_g^c). \quad (5.32)$$

The densities of the subtractive printing negatives, in terms of the densities of the separation negatives of the corresponding additive process, are given by the equations:

$$D_{cn} = D_{rn} - \gamma_{gc} D_{gn} - \gamma_{bc} D_{bn}, \quad (5.13)$$

$$D_{mn} = D_{gn} - \gamma_{rm} D_{rn} - \gamma_{bm} D_{bn}, \quad (5.23)$$

$$D_{yn} = D_{bn} - \gamma_{ry} D_{rn} - \gamma_{gy} D_{gn}. \quad (5.33)$$

The values of the coefficients which appear in Eqs. (5.13), (5.23), and (5.33) are given by:

$$\gamma_{gc} = (\gamma_{rn}/\gamma_{gn}) \cdot (V_r^m - V_b^m V_r^y)/(1 - V_b^m V_g^y), \quad (5.131)$$

$$\gamma_{bc} = (\gamma_{rn}/\gamma_{bn}) \cdot (V_r^y - V_r^m V_g^y)/(1 - V_b^m V_g^y), \quad (5.132)$$

$$\gamma_{rm} = (\gamma_{gn}/\gamma_{rn}) \cdot (V_g^c - V_b^c V_g^y)/(1 - V_b^c V_r^y), \quad (5.231)$$

$$\gamma_{bm} = (\gamma_{gn}/\gamma_{bn}) \cdot (V_g^y - V_g^c V_r^y)/(1 - V_b^c V_r^y), \quad (5.232)$$

$$\gamma_{ry} = (\gamma_{bn}/\gamma_{rn}) \cdot (V_b^c - V_b^m V_g^c)/(1 - V_r^m V_g^c), \quad (5.331)$$

$$\gamma_{gy} = (\gamma_{bn}/\gamma_{gn}) \cdot (V_b^m - V_b^c V_r^m)/(1 - V_r^m V_g^c). \quad (5.332)$$

Although there is no convenient photographic method for subtracting densities, the essential features required by Eqs. (5.13), (5.23), and (5.33) can be secured by the use of the masking method. If, for instance, positive transparencies (masks) are prepared from the green and the blue negatives, with gradients, γ_{gc} and γ_{bc}, then

the combined density of these two masks and the red negative, when superimposed, will be:

$$D_{cn}'=D_{rn}+(D_{gc}-\gamma_{gc}D_{gn})+(D_{bc}-\gamma_{bc}D_{bn}). \tag{5.14}$$

The quantities in parentheses are the densities of the masks, as given by Eqs. (4.23) and (4.33), for the conditions described above. The constants, D_{gc} and D_{bc}, are the values of D_{op} for the masks, from the green and blue negatives,

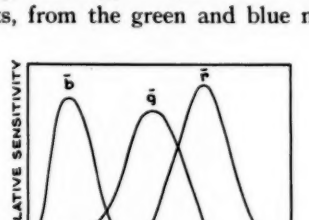

FIG. 7. Relative spectral sensitivities required for control of the intensities of the hypothetical projection primaries indicated in Fig. 6.

which are used in preparing the cyan printer. If similar masks are prepared, with the constants similarly defined, the densities of the masked green and blue negatives will be:

$$D_{mn}'=D_{gn}+(D_{rm}-\gamma_{rm}D_{rn})+(D_{bm}-\gamma_{bm}D_{bn}), \tag{5.24}$$

$$D_{yn}'=D_{bn}+(D_{ry}-\gamma_{ry}D_{rn})+(D_{gy}-\gamma_{gy}D_{gn}). \tag{5.34}$$

It is obvious from a comparison of Eqs. (5.14), (5.24), and (5.34) with Eqs. (5.13), (5.23), and (5.33) that the densities, D_{cn}', D_{mn}', and D_{yn}', of the masked negatives are simply the densities D_{cn}, D_{mn}, and D_{yn}, increased by certain constant densities. Therefore:

$$D_{cn}=D_{cn}'-D_{gc}-D_{bc}=D_{cn}'-D_{gb}, \tag{5.15}$$

$$D_{mn}=D_{mn}'-D_{rm}-D_{bm}=D_{mn}'-D_{br}, \tag{5.25}$$

$$D_{yn}=D_{yn}'-D_{ry}-D_{gy}=D_{yn}'-D_{rg}. \tag{5.35}$$

These values can be substituted for the quantities in the square brackets in Eqs. (5.11), (5.21), and (5.31), and it is evident from the resulting equations that the desired positive images, D_c, D_m, and D_y, can be printed from the masked negatives:

$$D_c=(D_o+\gamma_cD_{gb})-\gamma_cD_{cn}', \tag{5.16}$$

$$D_m=(D_o+\gamma_mD_{br})-\gamma_mD_{mn}', \tag{5.26}$$

$$D_y=(D_o+\gamma_yD_{rg})-\gamma_yD_{yn}'. \tag{5.36}$$

The quantities in parentheses are constants to which any desired values can be assigned, depending chiefly on the maximum color densities desired. The density, D_{gb}, is, in practice, simply the logarithm of the ratio of the exposure used in preparing the cyan printer (e.g., the wash-off relief matrix for the cyan dye) from the masked red negative to the exposure which would be necessary if a negative image having the theoretical densities, D_{cn}, could be prepared. The densities, D_{br} and D_{rg}, have similar effects on the exposures required in the preparation of the magenta and yellow printers.

It must be emphasized that the color reproduction secured with this masking method will be accurate only within the gamut of chromaticities for which the Eqs. (1.1), (1.2), and (1.3) predict the colors of subtractive mixtures with sufficient accuracy. This gamut is almost, but not quite, coextensive with the gamut of all colors attainable with mixtures of the three dyes. This is only a small, although the most important, fraction of the gamut enclosed within the triangle, *RGB*. The triangle *RGB* represents the gamut of colors accurately attainable with the theoretical additive process using the hypothetical primaries, *R*, *G*, and *B*. It should be unnecessary to mention that errors of reproduction will be caused by all departures of photographic and color densities from the linear portions of the density-*versus*-log-exposure curves.

E. Spectral Sensitivities of Color-Separation Negatives for Subtractive Color Reproduction

As stated in the first paragraph of Section D, the effective spectral sensitivities used in preparing the red, green, and blue separation negatives should be proportional to the tristimulus values of the spectrum transformed to the primaries, *R*, *G*, and *B*. These primaries for the wash-off relief process (shown in Fig. 6), were selected to give the best possible results when Eqs. (1.1), (1.2), and (1.3) were applied to the colors of the superimposed wash-off relief dye

images. It is convenient that these primaries are very saturated and that the triangle, *RGB*, includes all but a small portion of the entire gamut of physically realizable colors. As a consequence, the negative portions of the curves representing the transformed tristimulus values, $\bar{r}_\lambda$, $\bar{g}_\lambda$, and $\bar{b}_\lambda$, shown in Fig. 7, are quite negligible. The problem of attaining partially negative effective spectral sensitivities for the wash-off relief process can therefore be ignored in practice. As explained in the first paper,[1] it is not advisable to attempt to realize the secondary maxima of the red and blue sensitivity curves.

Figure 8 shows the relative spectral sensitivity curve typical of a type of panchromatic emulsion which is widely used and which can be obtained in all sizes of plates and films. Fig. 9 shows the transmittance curves of filters which would convert this emulsion sensitivity into the major positive portion of each of the effective sensitivities, $\bar{r}_\lambda$, $\bar{g}_\lambda$, and $\bar{b}_\lambda$, required for use with the wash-off relief process according to the theory here outlined. Filters having spectral transmittances nearly proportional to those shown in Fig. 9 are commercially available.[5] It is not considered feasible at present to design special filters which would have transmittances exactly

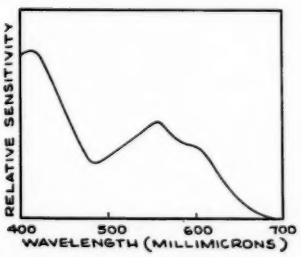

FIG. 8. Typical relative spectral sensitivity curve for an available panchromatic sensitization.

proportional to those shown in Fig. 9, because (1) the relative spectral sensitivity of any emulsion type is subject to considerable fluctuations from batch to batch, (2) the relative spectral sensitivity of any existing emulsion varies with the density of the image produced, and (3) the errors introduced by the existing filters having transmittances most similar to the curves shown in

[5] Eastman Kodak Company, "Wratten Light Filters," Rochester, 1938.

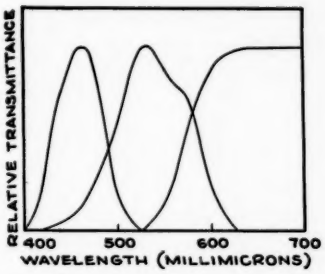

FIG. 9. Relative transmittances of filters necessary to convert the spectral sensitivity shown in Fig. 8 into the major positive portions of each of the desired spectral sensitivities shown in Fig. 7.

Fig. 9 are not greater in magnitude than the errors introduced by failures of the subtractive law, Eqs. (1.1), (1.2), and (1.3), and by nonlinearity of the density-*versus*-log-exposure characteristics of photographic materials.

For other sets of three-color dyes or pigments, the primaries, *R*, *G*, and *B*, may not be placed so advantageously. It may then be necessary to introduce the effects of partially negative spectral sensitivity in preparing the separation negatives. If this is accomplished by use of the masking method previously described,[1] then that masking method must be used, in general, before the masking method described in this paper can be applied. In other words, it may be necessary to use the masking method described in the first paper in order to prepare the color-separation negatives, having the correct densities, D_{rn}, D_{gn}, and D_{bn}. Such separation negatives must be prepared from three *principal negatives* combined with masks from one, two, or three auxiliary *correction negatives*. These principal and correction negatives result from suitably filtered exposures to the camera image of the original subject. After the correct separation negatives have been prepared by such a method, masks must be prepared from these in accordance with the constants defined by Eqs. (5.131), (5.132), (5.231), (5.232), (5.331), and (5.332) of the previous section, and each pair of these masks must be registered on the corresponding separation negative in order to secure the desired *printing negatives*.

In most cases, it will be sufficiently accurate to use the principal green negative as the green separation negative. The principal green negative

can also usually be used as the correction negative for the masking preparation of both the red and the blue separation negatives from the original red and blue negatives. Under these conditions it will be possible to combine into one process the two masking procedures, (1) for the introduction of the effects of partially negative spectral sensitivity, and (2) for rendering the subtractive color printing method equivalent to the additive projection method. Since, under these conditions, the principal negatives are the only negatives prepared in the camera, they will be called *camera negatives*.

The densities of the camera negatives will be represented by D_{rn}', D_{gn}', and D_{bn}'. Let γ_{lr} and D_{lr} represent the gradient and the maximum density of the mask from the green camera negative which is to be registered with the red camera negative to form the red separation negative. Let γ_{lb} and D_{lb} be similarly defined for the mask to be used in preparing the blue separation negative. The gradients, γ_{lr} and γ_{lb}, of the masks for introducing negative sensitivities must be computed from formulas given in the earlier paper.[1]

The densities of the resulting separation negatives are:

$$D_{rn}=D_{rn}'+D_{lr}-\gamma_{lr}D_{gn}', \tag{6.1}$$

$$D_{gn}=D_{gn}', \tag{6.2}$$

$$D_{bn}=D_{bn}'+D_{lb}-\gamma_{lb}D_{gn}'. \tag{6.3}$$

The gradients, γ_{rn}, γ_{gn}, and γ_{bn}, of these separation negatives, are:

$$\gamma_{rn}=\gamma_{rn}'-\gamma_{lr}\gamma_{gn}', \tag{6.11}$$

$$\gamma_{gn}=\gamma_{gn}', \tag{6.21}$$

$$\gamma_{bn}=\gamma_{bn}'-\gamma_{lb}\gamma_{gn}'. \tag{6.31}$$

In these expressions, γ_{rn}', γ_{gn}', and γ_{bn}', are the gradients of the camera negatives. If the densities of the separation negatives, given by Eqs. (6.1), (6.2), and (6.3) are substituted in Eq. (5.14) it will be found that the density of the cyan printing negative should be:

$$D_{cn}'=D_{rn}'-\gamma_{gc}'D_{gn}'-\gamma_{bc}D_{bn}'+D_{gb}', \tag{6.12}$$

where $$\gamma_{gc}'=\gamma_{gc}+\gamma_{lr}-\gamma_{bc}\cdot\gamma_{lb} \tag{6.13}$$

and $$D_{gb}'=D_{gb}+D_{lr}-\gamma_{bc}\cdot D_{lb}. \tag{6.14}$$

By similar substitutions, if the value of the magenta printing contrast, γ_m, given by Eqs. (5.22) is used, then the density of the magenta printing negative should be:

$$D_{mn}'=(1+\gamma_{rm}\gamma_{lr}+\gamma_{bm}\gamma_{lb})D_{gn}' -\gamma_{rm}D_{rn}'-\gamma_{bm}D_{bn}'+D_{br}', \tag{6.22}$$

where $$D_{br}'=D_{br}-\gamma_{rm}D_{lr}-\gamma_{bm}D_{lb}. \tag{6.23}$$

It is convenient to modify the printing contrast, γ_m, to the value γ_m', secured when the modified value of γ_{gn}, given by Eq. (6.211), is substituted in Eq. (5.22):

$$\gamma_{gn}=\gamma_{gn}'/(1+\gamma_{rm}\gamma_{lr}+\gamma_{bm}\gamma_{lb}). \tag{6.211}$$

When the modified value of the printing contrast is used, then the density of the magenta printing negative has the more convenient value:

$$D_{mn}''=D_{gn}'-\gamma_{rm}'D_{rn}'-\gamma_{bm}'D_{bn}'+D_{br}''. \tag{6.221}$$

The modified mask gradients, γ_{rm}' and γ_{bm}', are the values of γ_{rm} and γ_{bm} resulting from the substitution of the value, γ_{gn}, given by Eq. (6.211) in Eqs. (5.231) and (5.232). The constant, D_{br}'', has the value:

$$D_{br}''=D_{br}'/(1+\gamma_{rm}\gamma_{lr}+\gamma_{bm}\gamma_{lb}). \tag{6.231}$$

Finally, the density of the yellow printing negative should be:

$$D_{yn}'=D_{bn}'-\gamma_{ry}D_{rn}'-\gamma_{gy}'D_{gn}'+D_{rg}', \tag{6.32}$$

where $$\gamma_{gy}'=\gamma_{gy}+\gamma_{lb}-\gamma_{ry}\gamma_{lr} \tag{6.33}$$

and $$D_{rg}'=D_{rg}+D_{lb}-\gamma_{ry}D_{lr}. \tag{6.34}$$

Consequently, the three printing negatives which should be printed with the gradients, γ_c, γ_m', and γ_y, can be made directly from the camera negatives by superimposing, on each negative, masks made with the indicated gradients from the other camera negatives. A positive value of the gradient of any one of these masks signifies that that mask should be a positive image, and a negative value signifies that the mask should consist of a negative image. The particular values of the maximum densities, D_{gb}', D_{br}'', and D_{rg}' are not of critical importance, although it is desirable, for ease of registration and preparation of the printer, to prepare the camera negatives and the masks with the lowest values of these constants consistent with correct

tone reproduction. Masks whose computed gradients are less than 0.10 in absolute value may be omitted without serious error. Usually, only one of each pair of masks has a gradient greater than 0.10, so that the problem of registration becomes quite simple, and the technique reduces to the masking process which has been recommended and used frequently in the past.[3, 6] The derivation contained in this paper constitutes, therefore, a justification of the concept of masking, and the relations (5.12), (5.22), (5.32); (5.131), (5.132), (5.231), (5.232), (5.331), (5.332); (6.11), (6.211), (6.31); (6.13), and (6.33) specify in quantitative terms the correct gradients of the masks and of the printed images in terms of the gradients of the camera negatives.

F. Application of the Masking Method to the Wash-Off Relief Process

If camera negatives are prepared using emulsion and filter combinations resulting in the effective spectral sensitivities specified by the positive portions of the curves shown in Fig. 7, then these negatives can be used, without serious error, as the color-separation negatives suitable for use with the wash-off relief process. If these camera negatives are suitably exposed and developed, so as to result in uniform gradients of unity, $\gamma_{rn}=\gamma_{gn}=\gamma_{bn}=1.0$, then, by substitution of these values and the values given in Table V, in Eqs. (5.131), (5.132), (5.231), (5.232), (5.331), and (5.332), the gradients of the masks should be:

$$\gamma_{gc}=+0.42, \quad \gamma_{bc}=+0.00,$$

$$\gamma_{rm}=+0.58, \quad \gamma_{bm}=+0.09,$$

$$\gamma_{ry}=-0.02, \quad \gamma_{gy}=+0.43.$$

By similar substitutions in Eqs. (5.13), (5.23), and (5.33), the printing gradients should be:

$$-dD_c/dD_{cn}=\gamma_c=1.32,$$

$$-dD_m/dD_{mn}=\gamma_m=1.37,$$

$$-dD_y/dD_{yn}=\gamma_y=1.04.$$

Since γ_{bc} and γ_{ry} are practically zero, the corresponding masks may be omitted, without error. The value of $\gamma_{bm}=0.09$ is also low enough so that the corresponding mask may be omitted, but the error due to this omission may be appreciable for some colors.

The calculations exhibited above specify the use of a positive mask from the green camera negative to be registered with the red camera negative in preparing the cyan printer. This mask should have a gradient of 0.42 with respect to the green camera negative. Similarly, a positive mask from the red camera negative should be used in register with the green camera negative in preparing the magenta printer. This mask should have a gradient of 0.58 with respect to the red camera negative. Finally, a positive mask from the green camera negative should be used in register with the blue camera negative for the preparation of the yellow printer. This mask should have a gradient of 0.43 with respect to the green camera negative. For practical purposes, in this specific case, the same mask from the green camera negative can be used for the preparation of both the cyan and the yellow printers, since the respective gradients, 0.42 and 0.43, are practically identical. Thus, only two masks need be made, although each camera negative requires masking correction.

This method of correcting for the nonideal characteristics of the wash-off relief dyes has been tested and found to be quite successful. The actual colorimetric analysis of the accuracy of reproduction will not be published. The wash-off relief process has been used for illustration and test because the materials and technique are widely available. Consequently, the test can be repeated by all interested persons without the use of elaborate or expensive printing equipment. Some subtractive processes may not be so advantageous, because (1) the approximation method given for the prediction of the colors of subtractive mixtures may not be as satisfactory as in the case of the wash-off relief process, and (2) the primaries on which the most satisfactory approximation for any other subtractive process is based may not result in negligible negative portions of the theoretical spectral sensitivity curves. On the other hand, some processes of subtractive color reproduction may be more satisfactory than the wash-off relief process with respect to one or both of these considerations. No survey of available subtractive processes has

[6] Eastman Kodak Company, "Modern Masking Method of Color Printing," Rochester, 1937.

been attempted. However, it is reasonable to expect that the wash-off relief process, which was chosen for manipulative convenience alone, will be found to be neither the most nor the least advantageous process for the application of the masking method.

G. Summary

A rule for approximately predicting the colorimetric specifications of subtractive mixtures of certain sets of dyes has been developed. The analytical expression of this rule is sufficiently simple to permit its incorporation into the theory of color reproduction. This inclusion of the phenomena of subtractive mixture in the theory leads directly to the conclusion that masking is an essential part of any subtractive process of correct color reproduction. Formulas for the calculation of the gradients of the masks are given in terms of the parameters of the rule for the prediction of the colors of mixtures of the dyes to be used in printing the subtractive reproduction. Formulas are also given for the calculation of the gradients of masks to be used for the simultaneous introduction of partially negative spectral sensitivities and the compensations for the nonideal character of available subtractive printing colorants. An application of these formulas to the wash-off relief process shows that the theoretically desirable negative portions of the spectral sensitivities are negligible in magnitude, and that each separation negative should be printed in register with one mask. Since the masks specified for use with the red and blue separation negatives are practically identical, in the case of the wash-off relief process, only two masks need be prepared, one of which is used twice.

Acknowledgment

The author wishes to express his appreciation of the interest and assistance of Dr. M. W. Seymour, Mr. J. A. C. Yule, and of other members of the staff of these Laboratories, who suggested many of the lines of investigation featured in this and the previous paper.[1]

Theory of Subtractive Color Photography

II. Prediction of Errors in Color Rendering Under Given Conditions*

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IN Part I of this paper,[1] the requirements for obtaining perfect color reproduction were described. Since it is not usually practicable, in commercial processes, to satisfy all the requirements exactly, it is more important to be able to predict the errors in color rendering which will result from any given set of conditions. While such theoretical predictions must, of course, be checked experimentally, they will often eliminate the laborious testing of many cases which theory would have shown to be likely to give bad results.

There are at least three ways of expressing the errors in color rendering quantitatively; namely, in terms of the theory of colorimetry, for instance, as chromaticity and brightness errors; in terms of the density errors of the reproduction through the three color filters; or in terms of the errors of concentration of image components.

The first method has been applied by MacAdam[2a] and by Murray and Spencer[3] to additive processes, for which it is particularly suitable because of the ease with which the results may be worked out. This method will become much more valuable when data are available on the minimum perceptible differences of chormaticity in different regions of the color triangle. Even when such data are available, they must be used with caution, because it is unlikely that just distinguishable errors of hue, brightness, and purity are aesthetically of equal importance. At present, the colorimetric method has the disadvantages that workers in the field of color photography do not usually think in terms of colorimetry, and that if, in the case of subtractive color photography, the image components do not accurately obey any known laws of subtractive color mixture, the errors of color rendering can not be calculated theoretically. MacAdam,[2b] and Neugebauer[4] have worked out laws of subtractive mixture (the latter applying only to halftone processes) which may be used as a basis for the colorimetric approach to the problem.

The second method (determination of densities of a reproduction through color filters) is perhaps more suitable for the evaluation of actual practical tests.

For the reasons given above, and also because of the relative ease of calculation, the third method of expressing errors, i.e., in terms of the concentration of image components, will be used in the present paper.

It will be remembered that two conditions for obtaining perfect color rendering were formulated in Part I. These were as follows:

Condition 1: All colors whose spectrophotometric curves can be duplicated by suitable mixtures of the image components must be accurately reproduced.

Condition 2: All visually identical colors, whatever their spectral composition, must be reproduced by the same mixture of the image components.

In an imperfect process, the errors of color rendering may conveniently be divided similarly, according to whether they are due to failure to comply with Condition 1 or Condition 2.

ERRORS DUE TO NEGLECT OF CONDITION 1

It has already been pointed out that Condition 1 corresponds to the problem of making an accurate duplicate of a color photograph in the same medium, and that it may be again subdivided into two; i.e., making color separations

* Communication No. 685 from the Kodak Research Laboratories.

[1] J. A. C. Yule, J. O. S. A. **28**, 419 (1938).

[2a] D. L. MacAdam, J. O. S. A. **28**, 399 (1938).

[3] H. D. Murray and D. A. Spencer, Phot. J. **78**, 474 (1938).

[2b] D. L. MacAdam, J. O. S. A. **28**, 466 (1938).

[4] H. E. J. Neugebauer, Zeits. f. wiss. Phot. **36**, 22–89 (1937).

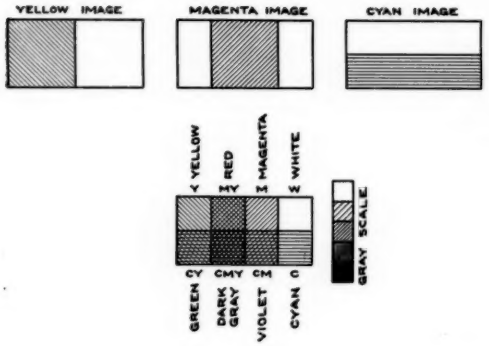

FIG. 1. Color chart for determination of errors in reproduction.

which are records of only one of the image components of the original, and obtaining an accurate reproduction of a scale of grays in the original. Throughout the paper it will be assumed that the contrast and density of image components are adjusted so that the reproduction of the gray scale is neutral in color but not necessarily a 45-degree straight line.

The method employed will first be described, and then its application to a few special cases commonly met with in practice will be discussed.

It is a relatively easy matter to determine how much of each image component will be present in the reproduction of any particular color of known spectral composition. This will give very little information about the accuracy of color rendering unless color mixture data are available for the set of dyes or pigments in use. By the simple device of making color separations from a specially prepared color chart described below, the desired information can be obtained without the necessity of following the tedious process of making actual color reproductions.

This color chart is made by printing uniform and partially overlapping patches of the image components of the process under consideration in such concentrations that the three when superimposed give an approximately neutral gray of density not less than 1, as shown in Fig. 1. Beside these patches is mounted a neutral gray scale.

The color chart will consist of eight patches which are designated by the initial letters of whichever image components are present. The colors of these patches will be yellow (Y), red (MY), magenta (M), violet (CM), cyan (C), green (CY), dark gray or black (CMY) and white (W). In the case of a halftone process in which the printing inks are printed as a solid in the chart, these eight colors correspond to the elementary colors (described by Neugebauer[4]) of which the halftone is composed.

The reproduction of a single color chart of this type gives a great deal of information about

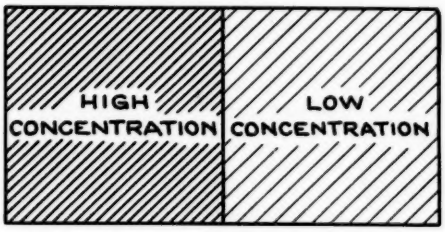

FIG. 2. Yellow image in general form of color chart.

the process in question; for more complete information a series of such charts could be made up, in each of which each image component would be printed in two different concentrations, as shown in Fig. 2.

Color separation negatives are made from this color chart, using the emulsions, filters, and illuminant which have been chosen for the color process under consideration. Provided they are not greatly underexposed, overexposed, or underdeveloped, the exposure and contrast of these negatives are of little importance. The gray scales of the negatives are plotted as shown for the blue filter negative in Fig. 3, and the densities of the color patches in the negatives measured and compared with the gray scale. Hence, the "color densities"* of the color patches through the different filters can be determined. For instance, the density of the MY (red) patch in the example under consideration was 0.40 in the blue filter negative; from the curve in Fig. 3 this is seen to correspond to a color density, or blue density of 0.93. All the color densities of

* The term "color density" will be used to denote the density of a region in the original through one of the color filters. Similarly, the density through the red filter will be called the red density. Color densities will be assumed to have been determined photographically, not visually, unless otherwise stated.

the eight patches can be determined similarly (see Table I) and plotted in a chart of the form shown in Fig. 4. If desired, the original color chart may be made from strips of the image components and made up in the form of a Lagorio chart (see Fig. 5). This method, by eliminating the effect of unevenness of the negative, will give more accurate results, provided the image components in the original are evenly printed. The relationship of this form of chart to that shown in Fig. 4 is easily seen.

It is sometimes difficult to understand why the color densities should be determined photographically rather than visually, unless the purpose for which they are used is kept in mind. The object of the color density tests is to determine how much of each image component will appear in the reproduction of each color. This obviously depends on photographic, rather than visual, sensitivity. This is made clear by taking the extreme case of using a colorblind emulsion with no filter, in the place of a panchromatic emulsion with a blue filter. The way in which a color would be reproduced would obviously bear no relation to the color density of the color measured visually with no filter. For instance, a pure yellow has a density below 0.05 measured visually, whereas, of course, it appears to the emulsion to have a high density.

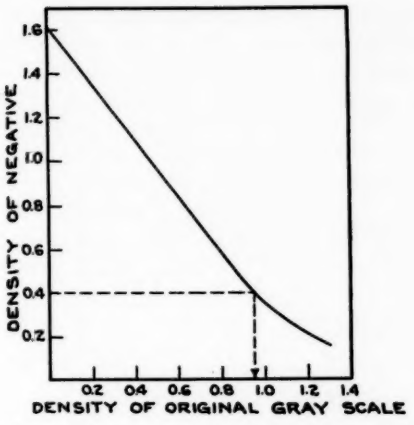

FIG. 3. Determination of color densities photographically.

TABLE I. *Color densities of color chart patches.*

1*	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	FILTER:	*B*	*G*	*R*	*G*	*B*	*G*		*G*	*G*	*B*	*G*	*R*	*G*
	Y(yellow)	0.89	0.08	0.01	0.08	1.45	0.13	0.07	0.14	0.04	1.03	0.08	0.01	0.13
	MY(red)	0.95	0.78	0.04	0.50	1.20	1.29	0.64	1.07	1.02	1.12	0.88	0.04	1.34
	M(magenta)	0.32	0.76	0.02	0.48	0.15	1.27	0.60	1.05	1.08	0.34	0.85	0.02	1.40
	CM(violet)	0.51	0.90	0.99	0.64	0.24	0.84	0.91	0.83	0.87	0.55	1.05	1.18	0.97
	C(cyan)	0.33	0.53	1.08	0.32	0.10	0.16	0.48	0.02	0.04	0.35	0.57	1.35	0.05
	CY(green)	1.00	0.58	1.08	0.34	1.20	0.24	0.50	0.11	0.06	1.20	0.63	1.35	0.15
	CMY(black)	1.05	0.95	0.88	0.72	1.12	0.90	0.94	1.02	1.00	1.29	1.12	1.02	1.18
	W(white)	0	0	0	0.06	0	0	0	0	0	0	0	0	0
	Errors in Color Densities													
	Y(yellow)	−0.16	+0.08	+0.01	+0.08	+0.33	+0.13		+0.14	+0.04				+0.13
	MY(red)	−0.10	−0.17	+0.04	−0.26	+0.08	+0.30		+0.05	+0.02				+0.16
	M(magenta)	+0.32	−0.19	+0.02	−0.28	+0.15	+0.28		+0.03	+0.08				+0.22
	CM(violet)	+0.51	−0.05	+0.11	−0.12	+0.24	−0.15		−0.19	−0.13				−0.21
	C(cyan)	+0.33	+0.53	+0.20	+0.32	+0.10	+0.16		+0.02	+0.04				+0.05
	CY(green)	−0.05	+0.58	+0.20	+0.34	+0.08	+0.24		+0.11	+0.06				+0.15
	CMY(black)	0	0	0	−0.04	0	0		0	0				0
	W(white)	0	0	0	0	0	0		0	0				0

* *Column*

1. Image components of color patches.
2. Approximate color of color patches.
3. } Color densities of color patches, with Wratten Filters Nos. 47, 58, and 25.
4. }
5. }
6. Printing densities of magenta ink, calculated from color densities in column 4 and reproduction curve in Fig. 7.
7. Corrected color densities obtained by combining 30 percent green and 10 percent red filter masks with the blue filter negative.
8. Corrected color densities obtained by combining a 40 percent red filter mask with the green filter negative.
9. Color densities obtained with an 80 percent green and 20 percent red filter exposure.
10. Corrected color densities obtained with a 55 percent red filter mask on a negative as in column 9.
11. Corrected color densities obtained with a negative as in column 9, a 53 percent green filter mask and a 10 percent blue filter mask.
12. } Color densities recalculated after subtracting 4 percent from reflectance.
13. }
14. }
15. Corrected color densities obtained from recalculated values, with a 40 percent red filter mask on a green filter negative.

Where the object is to compare the reproduction with the original by density measurements through color filters, the visual method is more logical. But, even in this case, the blue filter density will mean very little, since the brightness of the light transmitted through an ordinary blue filter has very little relation to its blueness. For example, it will be found that density through the Wratten C_5 filter gives little information about the intensity of two different yellows, unless they happen to consist of the same pigment. A wave-length of about 450 mμ gives the maximum intensity of blue sensation, whereas a bluish green wave-length of about 490 mμ appears to have the maximum brightness through a Wratten C_5 filter.

It would be possible, of course, to design a filter so that densities measured visually through it would indicate accurately the intensity of the blue sensation received by the eye, provided the "blue sensation" is defined by specifying a set of primaries. Such a filter would, however, be so dense that it would be impossible to use it in an ordinary densitometer.

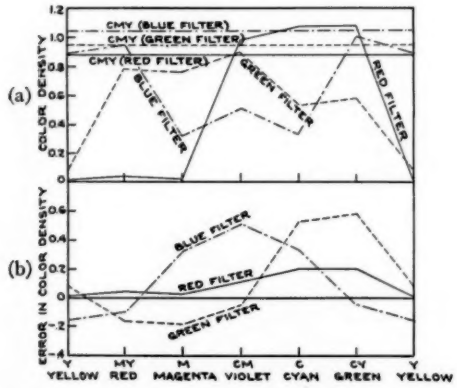

FIG. 4. (a) Graphical representation of color densities; (b) errors in color densities.

This objection does not apply to visual density measurements through red and green filters, since, in these regions of the spectrum, the visual sensitivity corresponds so closely to the photographic sensitivity of a panchromatic emulsion that the errors introduced are very slight.

The method of interpreting the color density data in terms of errors of color rendering will be described for certain cases met with in practice.

I. COLOR REPRODUCTION WITHOUT MASKING

A color chart of the form shown in Fig. 1 is made, and the color densities of the eight color patches through the three color separation filters determined as described. The amount of each image component which will print at any point in a reproduction of this chart will be directly related to the color densities of the patches, in a way which depends on the contrast and form of the reproduction curve of the process under consideration. For the moment, in order to eliminate the effect of errors in tone reproduction, it will be assumed that the gray scale is perfectly reproduced. In the original color chart, each one of the image components was present in only four of the eight color patches, being printed in equal concentration on all of the four. In a perfect reproduction of the color chart, the same would be true; *this can only be the case if, for each of the color separation filters, four of the color densities are high and equal, and four of them are equal to zero.** The extent to which the color densities depart from this requirement indicates the errors in color rendering resulting from the use of imperfect pigments and unsuitable spectral sensitivities. In estimating the errors in the color densities, it has been assumed that the CMY patch was correct in every case. This is not strictly true unless the CMY patch is neutral, but nevertheless it gives a useful indication of the magnitude of the errors. An example is shown in Fig. 4 which may be interpreted as follows: Considering first a single image component, the magenta, and assuming that a print were made in which the white and the CMY patches were correctly reproduced, it is evident from the chart that the M (magenta) and MY (red) patches have too low a color density and the C (cyan) and CY (green) too high a color density. In other words, in a reproduction of the color chart, the concentration of the magenta image component will be too high in the cyan and green and too low in the magenta and red. The actual values which these errors would have if the gray scale were accurately reproduced are shown in Fig. 4(b), obtained by

* The support on which the image components are printed should be taken as zero density, even though the white of the gray scale does not exactly correspond to it.

subtracting the ideal figures from the actual values of color density.

Considering all three image components in connection with the reproduction of any one color (for instance, the magenta patch), it will be seen that its red density is slightly too high, its blue density is much too high, and its green density slightly too low. Hence, in a reproduction, this magenta, and also magentas in general, will be somewhat less saturated and considerably too orange in hue. Individual magentas, pinks, purples, or crimsons may reproduce somewhat better or worse than is indicated by these figures. but in general, a uniform degradation of color in this direction will occur.

If the process is repeated using different color filters, or different image components, their effect on the color rendering may be compared. Strictly speaking, the process of color density determination should be repeated, using a wide range of different concentrations of the image components. However, the extra information obtained does not usually justify the work involved. A single test with the concentrations suggested is usually sufficient.

Since the tone reproduction curve of the final print, plotted against the original, is not usually a 45-degree straight line, the actual concentrations of image components can not be quantitatively determined from the color densities alone. If, however, a reproduction of a gray scale is made by the process under consideration, the color densities can be converted to so-called "printing densities," i.e., figures representing the actual amount of the image components in the reproduction. The printing density of a region in original, negative or reproduction is defined as the color density through the complementary filter of a single color reproduction made by the normal processing method. For instance, the magenta printing density of a certain color in the original is equal to the green filter density of that region in a magenta proof of a normal reproduction of the original. Fig. 6 shows a typical reproduction curve obtained for the magenta proof in the photoengraving process. It should be noted that, owing to the different amounts of the three image components required to make a neutral gray, the contrast, and even the shape, of the reproduction curve will be different for the three images. By the use of this curve, the color densities given in Table I are converted to printing densities which are given in Table I, column 4, and in Fig. 7. Since the reproduction curve chosen was not very different from a 45-degree straight line, these figures are not very different from the errors in color densities shown in Fig. 5. In fact, it is not usually desirable to carry out this last step in the calculations, as it gives little additional

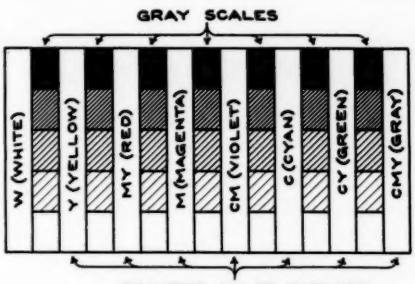

FIG. 5. Alternative form of color chart.

information, and merely serves to confuse the errors due to the tone reproduction curve with those due to the filters and image components. Very often (as in the present instance) the distortion of the reproduction curve appears to reduce the calculated errors in the printing densities. However, such an improvement is usually accompanied by a poorer rendering of some other colors not represented on the color chart.

Throughout this paper, errors have been represented in terms of density. The work of L. A. Jones[5] has indicated that, in a monochrome reproduction, gradient rather than density is the main factor in determining print quality, although this does not necessarily apply to the quality of color reproduction.

II. Results Obtained by the Masking Process

Without masking or retouching or the use of "ideal" image components, it is impossible to obtain perfect color rendering in a subtractive process. In Part I of this paper,[1] it was shown that perfection was at least a theoretical possi-

[5] L. A. Jones, J. Frank. Inst. **203**, 111 (1927).

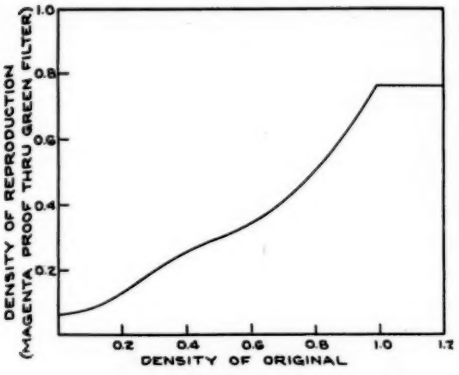

FIG. 6. Typical reproduction curve in photoengraving (direct).

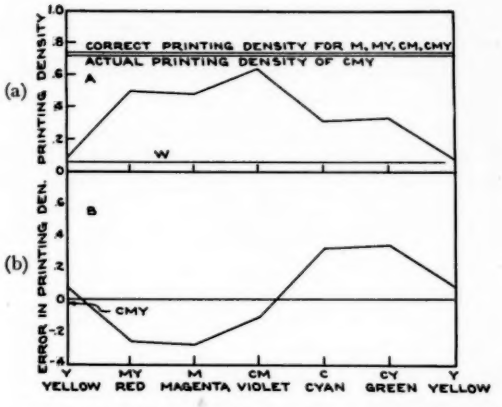

FIG. 7. (a) Printing density of magenta ink. Calculated from color densities in Fig. 4 and reproduction curve in Fig. 6; (b) errors in printing density.

bility by the use of the masking method. However, even when masking is used, it is not usually practical to obey all the requirements rigidly; and in the following discussion a method of evaluating a particular set of conditions is explained. This method is an extension of that described in the preceding pages.

The masking process has three effects: (1) The mask reduces the contrast of the negatives with which it is associated; this is overcome by using negatives of high contrast. (2) It increases the over-all density of the negative; to compensate for this a longer exposure is required in the next stage of the process. (3) Its most important effect is to alter the relationships between the densities of the various colors and the gray scale—i.e., to alter the color densities. In order to determine the effect of masking, it is only necessary to redetermine the color densities in the combination of mask and negative. The so-called "corrected color densities"* thus obtained may be interpreted as the color densities were interpreted in a straightforward, unmasked color reproduction. The "corrected color densities" may be determined by calculation from the uncorrected color densities, without actually carrying out the masking process.

The relatively simple case of negatives and positives whose characteristic curves are straight lines will be considered first. A color chart is made up exactly as before, and the color densities determined. It was shown previously that the density relative to white in a combination of masks and negatives is given by the formula:†

$$\Delta_c = c_1 D_1 + c_2 D_2 + c_3 D_3 + \cdots, \qquad (1)$$

where D_1, D_2, and D_3 are the color densities through the different filters.

For a neutral nonselective gray, the densities, D_1, D_2, and D_3, through the three filters must be equal ($=D_L$). Therefore,

$$\Delta = (c_1 + c_2 + c_3) D_L.$$

If D_L is the density of that step in the original gray scale which, in the masked negative, has the same density as the color patch under consideration, i.e., if D_L is equal to the corrected color density of the patch, then,

$$D_L = (c_1 D_1 + c_2 D_2 + c_3 D_3)/(c_1 + c_2 + c_3) \cdots. \qquad (2)$$

If

$$m_2 = c_2/c_1 \quad \text{and} \quad m_3 = c_3/c_1,$$
$$D_g = (D_1 + m_2 D_2 + m_3 D_3)/(1 + m_2 + m_3) \cdots. \qquad (3)$$

m_2 and m_3 will be negative in sign when a positive mask is used in conjunction with a negative, or a negative mask in conjunction with a positive.

* The "corrected color density" is defined as the density in the original, of that step of the gray scale which has the same density, in the masked combination, as the color patch under consideration.

† With partially diffusing images, this is only strictly true of diffuse densities. With some densitometers, slight discrepancies arise from the fact that the diffusing medium in the densitometer is not perfectly efficient.

The corrected color densities obtained by the use of this formula can be determined for all of the eight patches in the color chart, and used to plot a chart of the type shown in Fig. 4. Considering only the results for the yellow printer, we shall assume that a 30 percent green filter mask and a 10 percent red filter mask are to be combined with the blue filter negative. That is to say, m_2 and m_3 are equal to -0.3 and -0.1, respectively. This is not necessarily the most satisfactory choice of masks; it is merely given as an example of the application of the equation. The corrected color densities obtained with this combination are given in Table I, column 7. The masking is seen to raise those color densities which should be kept high (Y, MY, and CY), and reduce those (M, CM, and C) which require no yellow in the reproduction. The figures (derived from Eq. (3)) are given by the formula:

$$(D_B - 0.3D_G - 0.1D_R)/0.6.$$

The use of two masks in combination with a single negative is not common practice, although it is recommended in the "auto-masking" method for the wash-off relief process.[6] In this case, however, the blue filter negative is combined with a corrected green filter mask. Hence, the corrected green densities would have to be calculated first and used in the equation instead of the uncorrected green densities. The most common use of Eq. (3) is to determine what percentage mask will give the best results, after having decided tentatively what filters to use. For this purpose, when only a single mask is used with each negative, it is most convenient to plot the color density against the contrast of the mask, after which it can be seen at a glance what value is most satisfactory.

The method of plotting is shown in Fig. 8. The values of the color densities through the filters used for the mask and the negative are marked off as ordinates opposite the mask percentage values of -100 and 0, respectively. Corresponding points are joined by straight lines, and the corrected color densities for any percentage mask may then be read off directly. The values corresponding to a 40 percent mask are shown in Fig. 8. It will be remembered (see page 484) that in order to obtain perfect color reproduction, four of the color patches must have zero color densities, while four of them must be high and equal. No value for the mask satisfied these requirements, the best compromise being a 40 percent mask. Forty-nine percent, 54 percent, 19 percent, and 21 percent masks are required for the C, CY, MY, and M patches, respectively, taking the CMY patch as the high density standard. It is interesting to note that a 40 percent mask, which these calculations indicate to be the most satisfactory compromise, was recommended for these inks before the theory of masking had been worked out.

The reason for imperfect correction is that the inks do not obey the additivity law. The additivity law states that the color densities of the image components superimposed should be equal to the sum of the color densities of the individual image components. In the example given, the green density of the C, M, and Y patches are 0.53, 0.76 and 0.08, adding up to 1.37, whereas that of the CMY patch is only 0.95. This is caused by diffuse surface reflections, by the penetration of the ink into the paper, and by the fact that the presence of one ink on the paper interferes with the printing of the others.

In order to make the process more easily understood, an example is given in Fig. 9 of a case in which the yellow image component is perfect and the densities obey the additivity law. In such a case, almost perfect color correction is obtainable. It will be noted that an extension of the same diagram to the left shows the effect of a green filter mask on the red filter negative. In this hypothetical case such a mask is required. The graph may be regarded as a section of a three-dimensional chart from which the masking requirements for all three negatives could be read off. In this three-dimensional diagram, neutral grays would be represented by horizontal planes and all other colors by sloping planes.

A word about the method of expressing the contrast of masks is necessary at this point. In the past, they have been expressed as the ratio of the gray scale density range of the mask to that of the negative. In order to obtain straight lines in the diagrams just described, it was necessary to use a different scale of mask contrast, i.e., the ratio of the density range of

[6] *Making Color Prints on Paper from Kodachrome Films* (Eastman Kodak Company, Rochester, N. Y.), p. 5.

the mask to that of the combination of negative and mask.* The latter method has the additional advantage that when (as is usually the case) one is working for a definite density range in the combination, it facilitates calculation of the required density ranges of the mask and negative.

The foregoing is merely a graphical method of evaluating Eq. (3), the ordinates and abscissae being chosen in such a way that linear relationships are obtained. A knowledge of only two points on each line is then required.

It was noted that the color correction was imperfect because of the failure of the additivity law. Fortunately, it is possible to overcome this defect to a large extent by either of two methods: (1) by using negative materials with a specially shaped characteristic curve, or (2) by a suitable choice of filters (see Part I). The first alternative is not usually practicable. The second, on the other hand, offers a very convenient method of controlling the color densities.

In order to explain the principle, we shall confine ourselves to the green filter negative. By referring to Fig. 8, it will be seen that if a

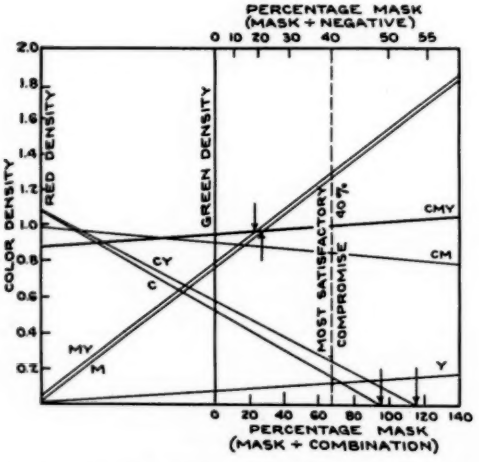

FIG. 8. Effect of masking on color densities, using a red filter mask with the green filter negative. The theoretical mask percentage required for each color is indicated by an arrow.

* For instance, with a negative of density range 1.95, and a mask of density range 0.65 (i.e., a 33 percent mask, according to the ordinary nomenclature), the combination will have a density range of 1.95–0.65, or 1.30. By the new method of defining mask contrast, this would constitute a 50 percent mask (i.e., $0.65/1.30 \times 100$).

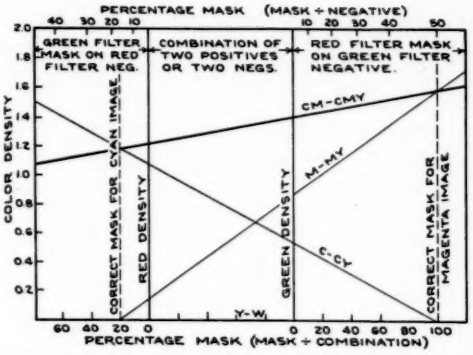

FIG. 9. Effect of masking on color densities, when color densities obey the additivity law.

strong enough mask is used to correct the CY and C patches, the color densities of the M and MY patches become much too high. Now, both these patches reflect red almost perfectly; hence, if the green filter used in making the color separation transmits a certain amount of red light, the color densities of these patches will be appreciably lowered. By redetermining the color densities, using filters of this type, it would be possible to choose a filter which would just correct these errors. However, the following is a much simpler method of determining the ideal filter.

Obviously, a similar effect could be obtained by giving the green filter negative a short supplementary exposure through the red filter. The effect of such an exposure can be readily calculated by converting the color densities to reflectances, and applying the following formula:

$$R_C = (E_G R_G + E_R R_R)/100,$$

where R_G = the reflectance of a color patch through the green filter, R_R = the reflectance of a color patch through the red filter, R_C = the reflectance corresponding to the combined exposures, and E_G and E_R = the percentages of the total exposure made through the green and red filters, respectively, corrected for any difference in filter factor.

Values of R_C obtained by the use of the formula are reconverted to densities, the new figures being used in the masking equations. These color densities can be more easily determined, however, directly from the type of

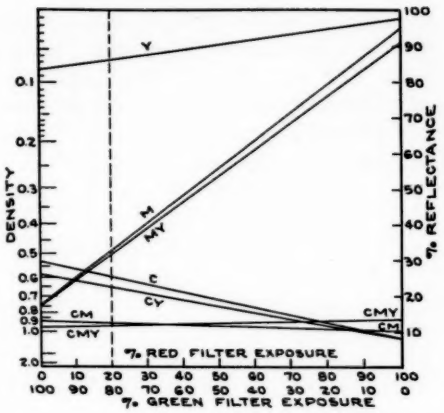

FIG. 10. Effect of supplementary exposure on color density.

graph shown in Fig. 10, in which the scale of ordinates is proportional to reflectance instead of density. Another graphical method of determining the effect of the supplementary exposure is shown in Fig. 11. The only color patches whose densities are greatly changed by the supplementary exposure are those whose density through the filter used for the supplementary exposure is much lower than that through the filter used for the main exposure. From Fig. 10 it may be roughly estimated that the best results will be obtained if the density of the *M* patch is reduced by about 0.3, corresponding to a supplementary exposure of 20 percent. If the filter factors of the two filters are unequal, the supplementary exposure must be increased or decreased in proportion to the ratio of the filter factors.

The effect of masking, using the new color densities obtained by the use of this supplementary exposure, is shown in Fig. 12. A 55 percent mask will now give the best results, and the results of this masking are shown in Fig. 13, curve 3, and in Table I, column 10. The results are seen to be much improved. A slight further improvement could be obtained by the use of a 10 percent blue filter mask with a 53 percent red filter mask, but the use of the second mask would not be justified in practice (see Table I, column 11, and Fig. 13, curve 4).

In general, the supplementary exposure should be made through the filter used for making the mask. It has been mentioned before, but may require further emphasis, that the supplementary exposure is only useful when aiming at the highest possible degree of color correction. When no masking, or masks of insufficient contrast, are used, the supplementary exposure will do more harm than good. Having determined the optimum supplementary exposure, the spectrophotometric curve of a single filter which would give the same effect can easily be calculated if desired. However, the split-exposure method is simple and more flexible.

III. Use of Negatives with Distorted Tone Rendering

The analysis just given is only applicable to the case where only the straight-line portion of the characteristic curve is used. By the use of "effective density" described in Part I, other cases can be analyzed. The details of this analysis will be described in a later paper. At the present time, it will be sufficient to mention that when masking is used, the distortion of the curve resulting from underexposure of the negatives results in much more serious errors of color rendering, while that resulting from overexposure improves the results and may render the supplementary exposure unnecessary, especially if these distortions of tone rendering are neutralized by some later stage of the process.

IV. An Acceptable Approximation

In many cases, the original subject contains colors outside the gamut of the reproduction process. Usually these are very saturated or very dense colors. By sacrificing some saturation

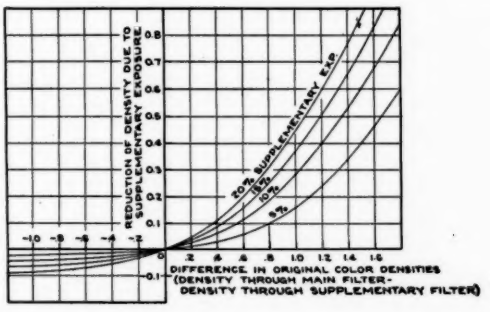

FIG. 11. Effect of supplementary exposure on color density (calculated).

and density, especially in the lower tones, a fairly acceptable approximation to an accurate reproduction can be obtained.

The method was suggested by the fact that a matte and a glossy print which differ only in gloss are considered approximately equivalent as regards color rendering. Under proper conditions of illumination, the only difference is that all points in the former reflect 4 percent more of all wave-lengths of light than the latter, the 4 percent being calculated as a percentage of the light reflected by a white region of the glossy print.

In the case of a process by which matte prints are obtained, the method consists in making a reproduction which would be perfect if it were varnished so as to give a perfectly glossy surface, i.e., if the diffuse surface reflections were eliminated. In other cases even though the final print does not have a matte surface, the object is to make a reproduction which, as regards the light reaching the eye of the observer under suitable illumination, bears the same relation to the original as a matte print does to a glossy, but otherwise identical print. Essentially, the method consists in a distortion of the tone reproduction, the shadows being flattened in the reproduction.

In order to determine the effect of this method, the color densities must be recalculated by converting them to reflectance, subtracting 4 percent, and reconverting to densities. It will then be found that the white patch of the color chart has a density of 0.02. Since the white is to be taken as having zero density, 0.02 must be subtracted from all the recalculated color densities. The results are given in Table I, columns 12, 13, and 14, and the effect of masking on these color densities is given in column 15. Apart from the loss of saturation in the lower tones which is inherent in this method, it will be seen that the residual errors after masking are much smaller than with the ordinary method (column 8). If the failure of the additivity law were entirely due to diffuse surface reflections, these errors would be entirely eliminated by this method.

It was mentioned above that the method involves a distortion of the tone reproduction. The required relation between the reflectance of the original and that of the reproduction is given by:

$$R_R=(1-S)(R_0+S),$$

where S = the diffuse surface reflection of a matte print (about 4 percent), R_0 and R_R = the reflectances of the original and the reproduction, respectively.

This distortion must be introduced after the masking has been carried out—for instance, in making the final print. If it is introduced in the negatives, e.g., by underexposure, followed by

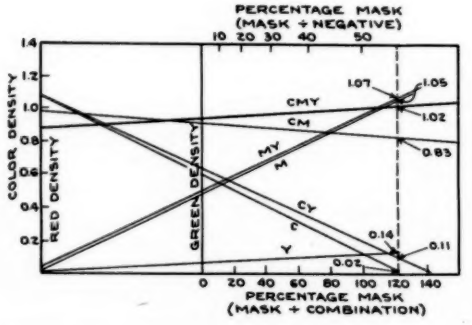

FIG. 12. Effect of masking after using supplementary exposure (combination of red filter mask with a negative made by an 80 percent green and 20 percent red filter exposure).

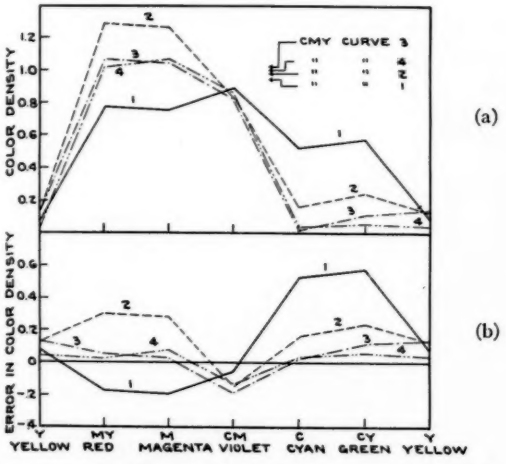

FIG. 13. (a) Color densities of magenta printer, under different conditions of masking; (b) difference between actual and ideal values. Curve (1)—unmasked; curve (2)—40 percent red filter mask; curve (3)—55 percent red filter mask, using supplementary exposure; curve (4)—53 percent red filter and 10 percent blue filter mask, using supplementary exposure.

combination of masks with the negatives, it will not give the desired result.

A reproduction made in this way, while it will usually be acceptable, will contain slight errors of hue as well as of saturation and brightness. This is due to the fact that certain colors (especially violet) are changed in hue by mere addition of white light.

When, as in many printing processes, the reproduction has a matte surface, the color densities may be determined experimentally instead of being recalculated. The color chart, with its gray scale, may simply be varnished with a clear colorless varnish, and the color densities redetermined, the new densities being used for calculation of the effects of masking. In practice, it has been found quite satisfactory to use such varnished color patches as a guide in making color separations.

Errors Due to Neglect of Condition 2

The methods just described tell us what errors of color rendering may be expected in attempting to duplicate a three-color original. They tell us, for instance, that if masking is not used, the reproduction of a magenta region in the original will contain too much yellow and not enough magenta, i.e., will be a rather pale orange-red. Furthermore, on the assumption that colors which are visually the same will reproduce approximately in the same way, we may infer (no longer confining ourselves to three-color originals) that reds, magentas, and purples, in general, whatever coloring materials they consist of, are likely to err in the same direction. However, until we have considered Condition 2 we can not predict exactly how, for instance, any particular red will reproduce, since its reproduction is likely to be influenced to some extent by the peculiarities of its spectrophotometric curve. A particular red will be much too orange in hue and too pale, owing mainly to the imperfections of the image components used in the reproduction; a further error, which may either aggravate or partially neutralize those already mentioned, may be introduced by the fact that the photographic emulsion, with its color filters, does not match colors exactly the same as the human eye.

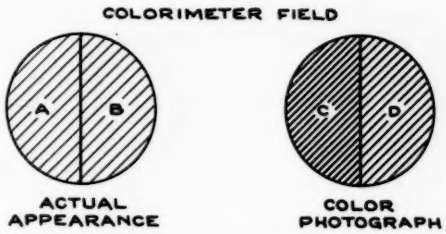

Fig. 14. Diagram of colorimeter field; A, sample; B, combination of dyed wedge matching sample; C, photograph of sample; D, photograph of dyed wedges.

It will be remembered that the second of the two conditions for obtaining accurate color reproduction was stated as follows: All colors which are visually identical must be reproduced by the same mixture of the image components. This condition will be fulfilled if the effective spectral sensitivities are linear transformations of the color mixture curves. The color filters commonly used do not satisfy this requirement. It is desirable to determine how serious the resulting errors are. A purely theoretical method, though it could be worked out, would be very laborious to use. A practical method of determining the errors, either in terms of color density or of actual color rendering, is described below. By this method, the errors resulting from failure to fulfill Condition 1 are eliminated, or at least do not interfere with the results.

A colorimeter is so arranged that the sample to be matched is seen in one-half the field, transparent wedges of the dyes used in the process under consideration being used for matching it. A camera is mounted so that the colorimeter field can be photographed. A series of miscellaneous colored objects, such as printed ink patches, dyed fabrics, flowers, leaves, etc., is chosen for test. One of the samples is matched in the colorimeter and a color photograph of the colorimeter field is made. This process is repeated for all the samples which have been collected. A miniature camera may be conveniently used for the purpose; in the tests actually made, the 35-mm color film was advanced by half a frame after each exposure. In studying a process which entails making separate color separations, exposures through each of the three color filters should of course be made for each sample, and

TABLE II.

	Color of Original Sample	Reproduction of Sample, Compared with Reproduction of Dye Wedges
103	Blood red	Both equally incorrect
113	Golden red	Sample decidedly better
202	Orange yellow	Sample very slightly better
206	Chrome yellow	Sample very slightly better
211	Lemon yellow	Sample slightly better
303	Light chrome green	Both equally incorrect
309	Emerald green	Sample very much worse
402	Cerulean blue	Sample very slightly worse
501	Royal purple	Sample slightly worse
503	Tyrian purple	Sample considerably worse
503	Claret	Sample slightly worse
	Flesh	Sample slightly worse
	Leaf green	Sample slightly worse

a neutral gray step tablet should also be photographed as a guide in processing.

In general, the two halves of the colorimeter field, although they were visually identical, will differ to some extent in the color photograph of the field. This difference is the result of neglect of Condition 2 only, and would not occur if the spectral sensitivities conformed to the requirements stated above. From results obtained in this way for a large number of samples, it can be decided whether the errors are serious enough to be worth correcting.

In addition to the differences between the two halves of the field, the color photograph of both halves is likely to differ markedly from the original in appearance. The situation is represented diagrammatically in Fig. 14. Referring to this diagram, field A matches field B, but C does not necessarily match D, and neither C nor D matches A and B. The difference between C and D is due to neglect of Condition 2, and depends on the form of the spectrophotometric curve of the sample A. The difference between D and B is due to failure to comply with Condition 1, and will be found to correspond to the errors predicted by the color density measurements described earlier. The difference between C and A represents the total error in the reproduction of sample A, and is, of course, the combined result of the two sources of error.

A number of printed ink samples were tested in this way, a commercial subtractive color film being used. The results are given in Table II. Since the areas of the fields in the color photographs were unfortunately not large enough for spectrophotometric tests, quantitative results can not be given. It was found that the differences between C and D were, in general, less than the differences between B and D (see Fig. 14)—that is to say, in the process studied, the errors resulting from the neglect of Condition 1 were much more serious than those resulting from the neglect of Condition 2. Nevertheless, the latter errors were by no means negligible, being mainly due to errors in the concentrations of the cyan, and to a smaller extent, the yellow dyes. This is to be expected, since the red sensitivity of the film has a maximum at about 650 mμ instead of the theoretically required 600 mμ, while the blue sensitivity extends into the ultraviolet.

SUMMARY

By the use of a special form of color chart, the errors in color rendering resulting from any given combination of dyes and filters can be predicted. The effect of any combination of masks can also be calculated. A method of improving the results by means of partial exposures through two color filters in making the color separation negatives is described. A method of determining the errors due to the fact that spectral sensitivities fail to correspond to the color mixture curves for the human eye is described.

The Astigmatism of the Concave Grating as a Means of Calibrating Photographic Plates in Intensity Measurements*

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The intensity of the astigmatic spectral image formed by a concave grating is a maximum at the center and drops off to zero at the ends of the line. Inasmuch as the intensity distribution can be known from the value of the astigmatism factor, G. H. Dieke has suggested using such an image as a means for calibrating a photographic plate in intensity measurements. It has been the purpose of this investigation to develop the method in detail, to apply it to the calculation of relative intensities, and to compare the results with those obtained from one or another of the standard methods now in use. Under proper conditions the astigmatism method has the great advantage of simplicity, and it was found to yield results which compare favorably with those obtained by other means.

G. H. DIEKE[1] has suggested using the astigmatism of the concave grating for intensity measurements, as follows: It is known that the intensity of an astigmatic spectral image is a maximum at the center and tapers off to zero at the ends of the line; and furthermore, the intensity distribution along the length of the line can be determined from the value of the astigmatism factor. If the image is photographed, a blackening corresponding to this varying intensity is produced on the plate, and the variation can be found by running the line lengthwise through a microphotometer. Since the calibration of a photographic plate consists in finding what blackenings are produced by a series of light sources of known intensities, the astigmatic image supplies all the needed data for the construction of a calibration curve, and therefore forms the basis of a simple method for use in intensity measurements.

The object of the investigation here described was to study the conditions governing this method, to apply it to the calculation of intensities, and to compare the results with one or another of the standard methods now in use.

THEORY

By reason of the astigmatism of the concave grating, a point source of light on the Rowland circle will give a spectral line of length[2]

$$\zeta L = (\sin^2 \beta + \sin \alpha \tan \alpha \cos \beta) L,$$

in which α and β are the angles of incidence and diffraction of the light beam. ζ, the astigmatism factor, is the ratio of the image length to the length of the grating ruling, L. When a line source is used, such an image is formed for every point along its length. These images are partially superimposed in such a way that the center of the image due to the lowest point on the slit lies at a distance $s/2 \cdot \cos \beta / \cos \alpha$ above the Rowland circle, s being the length of the slit. Hence when a slit of some length is used, the total length of the image is given by[3]

$$\zeta L + s \cos \beta / \cos \alpha.$$

If it is assumed that the intensity of a point source is uniformly distributed along ζL, then the slit image has a central portion of constant intensity, the length of which depends on the relation of the slit length, s, to the length of the astigmatic image of a point source. The relative positions of the images coming from the successive points along the slit are shown in Fig. 1, (a), (b), and (c), where s is equal to, greater than, and less than $\zeta L \cos \alpha / \cos \beta$, respectively. It is seen that in every case the length of the constant portion is given by[3]

$$\zeta L - s \cos \beta / \cos \alpha.$$

* In partial fulfillment of the requirements for the degree of Doctor of Philosophy at the Catholic University of America.

[1] G. H. Dieke, J. O. S. A. 23, 280 (1933).

[2] Kayser, *Handbuch der Spektroskopie*, Vol. 1, p. 463.

[3] Reference 2, p. 464.

FIG. 1. The astigmatic lines from individual elements of slit.

The intensity of this constant portion is a function of the angles of incidence and diffraction, and can be shown to have the following values:

$$I_{max} = I_0 \cos \alpha / \cos \beta \quad \text{when} \quad s \geqq \zeta L \cos \alpha / \cos \beta,$$

$$I_{max} = I_0 s / \zeta L \quad \text{when} \quad s < \zeta L \cos \alpha / \cos \beta,$$

where I_0 is the intensity covering the grating per *unit length* of slit, and I_{max} is the intensity per *unit length* of image in the constant portion of the line. Losses due to reflection and other causes in the instrument are neglected.

Uniform intensity along the length of ζL also results in a linear distribution of intensity from the central constant portion to the end of the line. On this account, when the length of the constant portion and the total line length are known, a simple equation can be written which gives the relative intensity at any point between the end of the line and the constant central part. This equation is, as seen in *ab* of Fig. 2, $y = mx$, where the slope, m, is $I_{max}/\zeta L$ or $I_{max}/s(\cos \beta/\cos \alpha)$, depending on whether $s \cos \beta/\cos \alpha$ is greater or less than ζL. If the two expressions are equal, either form applies. x, of course, lies between zero and ζL or $s \cos \beta/\cos \alpha$, and y is the corresponding intensity. When an astigmatic line of this kind is scanned by the luminous spot of a microphotometer and the galvanometer deflections are photographically recorded, a curve is obtained in which the deflections can be made ordinates and the distances from the end of the line, abscissae. To every point on this curve there corresponds a point on the intensity curve, so that it can be represented by *ABCD* of Fig. 2, and the two combined, therefore, serve as a calibration curve. Their use as such will be discussed later.

In order to obtain an astigmatic image with intensity distribution as described, three conditions must be fulfilled. First, every ruling of the grating must be such that every point along its length contributes evenly to the intensity of the line,[1] otherwise ζL will not possess the required uniformity, and the distribution along the resulting image cannot be calculated in a simple manner. Second, the grating must be filled vertically with light in a uniform manner. Therefore here, as in practically all single-exposure photographic methods, a source uniform in space is required. Third, it is necessary to adjust the slit to exact parallelism with the rulings of the grating to insure perfect superposition of the ζL's resulting from the successive points along the slit.

To scan a line along its length successfully with a microphotometer spot,[1] the spectrum must be photographed with a fairly wide slit. If the lines are so closely located that this cannot be done, then a continuous spectrum can be photographed in the same region and on the same plate as the lines to be measured. By running across this spectrum at the wave-length desired, data can be obtained for a calibration curve.

DESCRIPTION OF APPARATUS

The spectrograph used for the work is the Eagle mount in the Spectroscopy Laboratory at the Catholic University of America. The concave grating has a radius of curvature of 5 meters, a ruled surface 6 inches long, and an effective height of 5.4 cm. The astigmatism of the grating ranges from zero at the central image to approximately 0.6 at third-order red. Eastman spectroscopic plates, Class F, Types II and III, were used for all exposures.

A calcium arc in air and a neon tube were the sources used for the intensity measurements. The calcium arc was obtained by introducing calcium into a carbon arc. A 220-volt supply was used, with sufficient resistance in the circuit to cut down the potential across the electrodes to about 60 or 70 volts. The current averaged 6.5 amperes in all trials made. The neon lamp was a Cooper-Hewitt hot cathode type, operated on a 110-volt a.c. source through an auxiliary unit A potentiometer arrangement was used to keep the current constant. As a source for the production of a continuous spectrum an ordinary 6-volt Tungsten Ribbon National Mazda lamp was used. The potential across it was kept con-

stant by means of a variable resistance in series with it.

The microphotometer employed was the Kipp and Zonen instrument in the Department of Organisms and Radiation of the Smithsonian Institution. Calibrated filters, obtained from the Department of Colorimetry at the National Bureau of Standards, were used to check the results obtained by the astigmatism method.

Experimental Procedure

Before undertaking any intensity measurements, it was necessary to investigate the extent to which the grating satisfied the first condition mentioned above, namely that of uniform reflectivity along the grating rulings. This was done in the following manner: A very short slit, approaching a point, was used with a source constant in space, and the resulting line image was photographed and run through the microphotometer. On account of the point-to-point relation between grating and image, the intensity distribution along the length of this line corresponded to the reflectivity of the grating, not along the separate rulings, but along all the rulings taken collectively. Approximately the central third was constant. This was interpreted as meaning that the horizontal summation along the vertical central third of the grating was the same as if every ruling in that section, considered independently, was perfect. It was concluded, therefore, that this portion of the grating would satisfy the condition set down in the theory. The process was repeated for several mercury, sodium, and neon lines. Though the lines were of different lengths on account of the varying astigmatism, all gave the same general result. To be certain that the reflectivity of the grating was the only influencing factor, photographs were taken with and without a lens between source and slit, with the lens in various positions, and with slight changes in the size of the "point." The plates always showed the same intensity distribution along the length of the line images. A central section of the grating corresponding to the constant portion of one of these microphotometer records was used in all the intensity measurements made; it had a vertical height of 15 mm.

The calcium lines

The group of calcium lines chosen included Ca 5581.97, 5588.74, 5590.11, 5594.46, 5598.48, 5601.28, and 5602.83 in the third order where the value of the astigmatism is approximately 0.48. In an arc of the type used spatial constancy cannot be expected. A diffusing screen of ground glass was therefore employed to secure the required evenness of illumination. Care was taken to exclude the light from the poles. The spectrograph slit was 0.5 mm wide and 1 cm long. Ca 5588.74, having the greatest intensity, was used as a calibration line; and 5594.46, next in order, was used in the same capacity for the remaining lines. In this way there was a check on five of the lines in the group. The calibration lines were scanned lengthwise by the microphotometer spot, and the whole set was crossed at right angles. In this manner the ratio of the intensities of the lines was determined. Details of this evaluation will be described later.

The same lines were measured again with a tungsten lamp, possessing the required spatial constancy, as calibrating source. The continuous spectrum was photographed on the same plate and in the same wave-length region as the calcium lines to be measured. The microphotometer spot was made to scan the tungsten

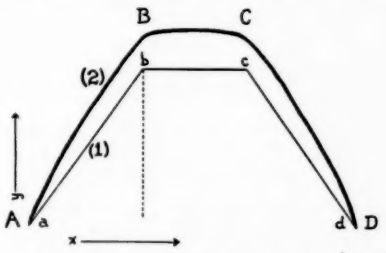

Fig. 2. (1) Ideal intensity curve, and (2) microphotometer trace for the astigmatic line along its length.

spectrum from top to bottom in the region of 5590, and to cross the whole set of calcium lines at right angles. The variable slit-width method was employed to check the results obtained in the two measurements just described. Exposures for this and the autocalibration astigmatism method were all made on the same plate to minimize errors due to plate variation.

A group of red calcium lines, 6439.09, 6462.58, 6471.66, 6493.79, and 6499.65, in the second

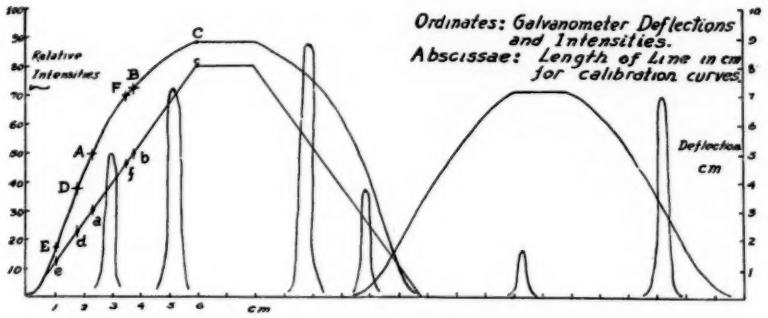

FIG. 3. The intensity curve, and actual microphotometer record for neon lines.

order, was handled in the same way. The results obtained in all these tests are separately listed.

The neon lines

Six lines in the neon spectrum, 6074.337, 6096.162, 6143.061, 6163.594, 6217.279, and 6266.495, were next chosen for measurement. This region was homochromatic for the plates used within 2 percent, according to spectral sensitivity data obtained from Eastman. Allowance for a small heterochromaticity is not necessary, however, to *check* an intensity method, provided the same kind of plates are used throughout. The neon tube was very long; therefore it was reasonable to assume that a small portion of the glow might be constant. An image of such a small portion was focused on the slit of the spectrograph and the lines, in the third order, were photographed; a 0.5-mm slit 1.2 cm long was used. The perfect symmetry of the resulting microphotometer graphs justified the assumption of constancy in the source. Ne 6143.061 and 6096.162, the lines of highest intensity, were used as calibration lines, and in this way a check was had on most of the lines in the group. Several exposures were made. The results thus found were checked by the calibrated filter method.

Interpretation of the Microphotometer Records in the Astigmatism Method

After the microphotometer record has been obtained, the first step in the interpretation is the calculation of the length of the calibration line and of its constant portion from theory. By using the four points thus determined a theoretical intensity curve can be drawn. Fig. 3 records the deflections produced by crossing the six neon lines at right angles and also the result obtained by running the microphotometer along the length of the two calibration lines. In this region, for $\lambda = 6143$, third order, $\alpha = 31° 13' 12''$, $\beta = 34° 45'$, and $\zeta L = 0.874$ cm. The calculated lengths multiplied by the magnification factor of the microphotometer, which is 6.72, are the following:

Length of line $= 13.62$ cm
Length of constant portion $= 1.87$ cm.

With this data the intensity curve was constructed and fitted under the microphotometer trace. The galvanometer deflections due to the lines correspond on the blackening curve of Ne 6143.061 to points A, B, C, D, E, and F which project points a, b, c, d, e, and f on the intensity curve. The relative intensities of the lines can be read directly from the ordinates of these points. The results, in terms of 100 for the strongest line, are given in Tables I and II.

Discussion of Results

In judging any particular photographic method for measuring intensities, it must be remembered that there are errors inherent in the photographic method as such. These are mainly: Errors caused by variation in plate sensitivity, errors due to developing, background difficulties, and errors in the microphotometry. It is generally conceded that unless a large number of measurements is made, the error due to these sources is about 3 percent. It is difficult in a given case to say just what part of the error is ascribable to these

sources, and what part to the specific method in question. Nevertheless, the results obtained by the astigmatism method are, with a few exceptions, reasonably self-consistent.

In comparing the astigmatism method with other methods, some difficulty was experienced in the measurement of the calcium lines. This was because of the unsteadiness of the arc, which cut down the exposure time by an unknown factor. Unless autocalibration methods can be employed, this is a serious source of error and probably accounts to some extent for the discrepancy between the variable slit width and astigmatism methods. The neon lines did not present this difficulty, and the results obtained by the astigmatism method compare favorably with those derived from the use of the calibrated filters. It may be mentioned here that this paper is concerned with only the effective relative intensities of the lines.

Sources of Error in the Method

The main sources of error in the astigmatism method itself are: first, diffraction; and second, the discrepancy between the actual and calcu-

TABLE I. *Effective relative intensities of the calcium lines.*

	5581.97	5588.74	5590.11	5594.16	5598.48	5601.28	5602.83	
	18.9	80*	21.1	58.8	42.1	21.1	21.1	Plate 1
Astigmatism	21.1	80*	18.9	55.2	42.9	21.9	18.7	Plate 2
Autocalibration	20.9		19.1	55.2†	43.2	22.0	19.7	
	20.9	80*	19.6	56.3	43.4	22.5	19.6	Plate 3
	20.2		19.6	56.3†	42.8	21.9	19.6	
Average	*20.4*	*80*	*19.7*	*56.4*	*42.9*	*21.9*	*19.7*	
Variable Slit-width	21.8	72.5	22.1	56.4	44.7	24.2	22.6	

	6439.09	6462.58	6471.66	6493.79	6499.65	
Astigmatism	150*	129.9	39.0	82.5	30	Plate 4
Autocalibration	150*	130.8	37.5	80.5	31.9	Plate 5
		130.8†	34.3	81.7	29.4	
Average	*150*	*130.5*	*36.9*	*81.6*	*30.4*	
Astigmatism	150	126.6	40.3	82.0	33.3	Plate 6
W for calibration	150	126.5	40.7	81.5	31.5	Plate 7
Average	*150*	*126.5*	*40.5*	*81.8*	*32.4*	
Variable Slit-width	150	130.0	41.0	83.9	28.8	

* Calibration line.
† Second calibration line on same plate.

TABLE II. *Effective relative intensities of the neon lines.*

	6074.337	6096.162	6143.061	6163.594	6217.279	6266.495	
	35.0	61.1	100*	29.9	19.3	58.9	Plate 1
	35.8	61.1†		29.6	20.1	59.3	
Astigmatism	36.6	60.2	100*	30.7	19.4	57.3	Plate 2
Autocalibration	36.4	60.2†		31.2	20.8	57.6	
Method	37.4	61.1	100*	29.5	18.0	56.7	Plate 3
	38.2	61.1†		29.9	18.8	57.7	
	35.7	61.1	100*	28.2	17.9	54.1	Plate 4
	38.9	61.1†		28.1	18.6	53.2	
Average	*36.8*	*60.9*	*100*	*29.6*	*19.1*	*56.9*	
Calibrated Filter Method	38.7	61.5	100	28.9	18.1	52.1	

* Calibration line.
† Second calibration line on same plate.

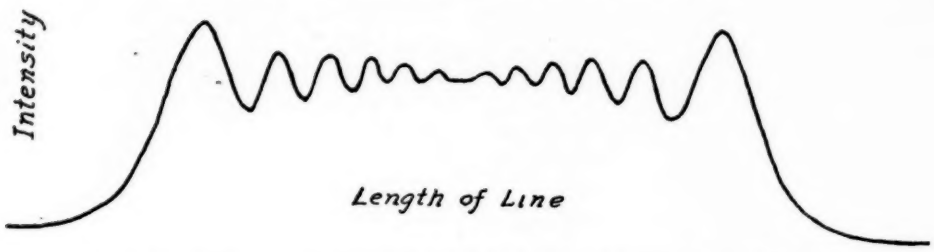

FIG. 4. Theoretical intensity along astigmatic line from a point source due to diffraction.

lated length of the astigmatic image and its constant portion.

In the theory underlying the use of astigmatism as a means of plate calibration it was assumed that light coming from a point source is uniformly distributed along the length of the line image produced by the grating. During the course of the work, however, it was found that the assumption is not entirely justifiable. A photograph of the image of a point source shows a very definite diffraction pattern along its length. This pattern resembles that of two straight edges, and appears whether the whole or a part of the grating is used. To study the situation the pattern formed by two straight edges placed midway between a point source and a screen, and 435 cm* from each, was worked out by the Cornu spiral method, and when the distances between maxima were multiplied by the astigmatism factor for the third order of 5890 they corresponded with those measured on the photographic line image of a point source in that region. Fig. 4 represents the intensity distribution along the length of the line as obtained from the Cornu spiral calculation. This feature practically does not affect the linear distribution of energy along the astigmatic image of a slit, for an analysis showed that when a large number of intensity patterns like that in Fig. 4 are superimposed as are the uniform lines in Fig. 1, the maxima and minima are smoothed out. The only slight change (Fig. 5) is a curving off at the ends of the central constant portion, and a "tail" that curves away at the end of the line.† It follows that an appreciable error is introduced if the intensity of the line to be measured lies within two or three percent of the calibration line. The difficulty can be obviated, however, by measuring the blackening of the lines at some constant distance from the central portion.

The second source of error, the discrepancy between the actual and calculated lengths of the astigmatic image and its constant portion, depends on the calculation of ζL. If $s \cos \beta / \cos \alpha > \zeta L$, an error in the calculation of ζL introduces an error into the length of the line and its constant portion, and therefore into the slope of the intensity graph. It is estimated that an error of 1 percent in ζL would thus produce an error ranging from ∓ 2 percent in the weakest to ± 2 percent in the strongest lines. If $s \cos \beta / \cos \alpha < \zeta L$, an error in the latter quantity will cause the actual intensity curve to be parallel to the theoretical one, but displaced by a small amount. This introduces a constant error throughout the intensity range, also of about 2 percent if ζL is out by 1 percent. The error in ζL probably does not reach 1 percent, for L can be measured as accurately as desired and ζ depends only on the angles of incidence and diffraction which can be known within a fraction of one percent.

CONCLUSION

The astigmatism method of plate calibration has the very great advantage of simplicity for two reasons: First, it calls for no auxiliary apparatus to produce the intensity variation required for calibration; and, second, it dispenses with the labor involved in the construction of the usual characteristic curve. It is true that certain calculations must be made to enable one to use the microphotometer record obtained by scanning the calibration line, but these

* Distance between slit and grating for third order of 5890.

† This probably accounts for the appearance of "tails" at the ends of astigmatic images when long exposures are made.

calculations need be made only once in a given region, and, provided the setting of the instrument for that region is always the same, can be used in as many trials as might be desired.

The question arises as to the type of mount which is desirable. Certainly that one is best suited to the method in which the astigmatism is greatest and therefore a Paschen mount would be preferable to the Eagle mount used in this investigation. If the astigmatism is great, ζL will be large, and the slope in the equation, $y=mx$, will be small. Both conditions tend to render the method more sensitive.

FIG. 5. Actual intensity distribution along composite astigmatic line.

The greatest difficulty associated with the method is that of obtaining the required grating conditions. Many of the older gratings do not reflect evenly, yet a study of any particular one will probably reveal a portion of the grating which is effectively good. At the present time, however, more uniform gratings are available, and their use should greatly simplify the application of this astigmatism method.

In conclusion the writer wishes to express her deep gratitude to Dr. Carl A. Beck of the Catholic University of America who, in directing this investigation, not only evinced real interest and contributed many helpful suggestions, but also rendered invaluable assistance throughout the work. She wishes likewise to acknowledge her indebtedness to Professor Karl F. Herzfeld; to Dr. E. S. Johnston of the Smithsonian Institution for the use of the microphotometer in the Organisms and Radiation Laboratory; and to Dr. K. S. Gibson of the National Bureau of Standards for the loan of some calibrated filters.

Selected Ordinates for Computing Trichromatic Coefficients and Candlepower of a Light Source

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(Received September 28, 1938)

MOST problems in colorimetry are concerned with the determination of the color qualities of a material when illuminated by a standard light source, such as Illuminant C of the International Commission of Illumination. For this reason, much is to be found in the literature to aid the worker in this field, the *Handbook of Colorimetry*, prepared by the staff of the color measurement laboratory of Massachusetts Institute of Technology, under the direction of Dr. A. C. Hardy, being an outstanding example.

The tables of selected ordinates given in that book are especially helpful in minimizing the labor involved in the reduction of spectral-reflection or spectral-transmission data for a sample illuminated by one of the standard illuminants of the I.C.I. So far as the writers are aware, however, no selected ordinate tables have ever been published for use in the evaluation of the light source itself, and it is the purpose of this paper to present such a table for the use of those interested in performing these calculations.

The procedure to be followed in the use of Table I in the calculation of the tristimulus values of a light source is as follows. It is assumed that the spectral-energy distribution curve of the light source is known, and that the energy ordinates are based upon a unit band width of 10 millimicrons (we have specified our light sources by plotting irradiance of a surface at unit distance from them and have expressed irradiance in microwatts per cm² per 10 millimicrons). The ordinates of the distribution curve at each of the thirty wave-lengths listed in the X column of the table are first determined. These are then added and multiplied by the constant 0.3561, to give the X coefficient of the source. Similarly, the Y and Z coefficients are obtained by adding the ordinates at the thirty wave-lengths listed in each case, and multiplying these sums by 0.3562 and 0.3559, respectively.

The wave-lengths of the thirty ordinates for each coefficient were determined in the following manner. First, using the X value as a typical case, the area under the curve showing the variation in the $\bar{x}$ tristimulus values of the spectrum colors with wave-length was divided into thirty equal parts. The thirty ordinates which divide these thirty equal areas in half were then determined, these constituting the selected ordinates for the X coefficient.

The integration thus proceeds in steps of equal stimulus, instead of steps of equal wave-

TABLE I. *Thirty selected ordinates for computing the trichromatic coefficients of a light source.*

Ordinate Number	(X)	(Y)	(Z)
1	423.7 mμ	469.7 mμ	410.4 mμ
2	436.5	493.1	419.6
3	446.8	504.6	424.1
4	457.7	512,1	427.4
5	472.1	518.0	430.1
6	531.6	522.9	432.5
7	544.9	527.4	434.8
8	553.4	531.6	437.0
9	560.0	535.5	438.5
10	565.5	539.3	441.1
11	570.4	543.1	443.1
12	574.8	546.7	445.1
13	578.9	550.3	447.1
14	582.8	553.9	449.1
15	586.4	557.4	451.1
16	589.9	561.0	453.2
17	593.4	564.6	455.2
18	596.7	568.3	457.2
19	600.1	572.0	459.3
20	603.5	575.9	461.5
21	606.9	579.9	463.7
22	610.4	584.1	466.0
23	614.0	588.5	468.5
24	617.9	593.3	471.3
25	622.1	598.5	474.3
26	626.7	604.3	477.9
27	632.2	611.0	482.4
28	638.8	619.1	488.2
29	648.0	629.9	496.9
30	665.4	649.7	513.7
Multiplying Factors	0.3561	0.3562	0.3559

length, and the average of the energy ordinates so selected is the ordinate of an equal-energy curve having the same X coefficient as that of the source in question. Now the X coefficient of a source having an equal-energy distribution of one unit of energy per 10 millimicrons is equal to 10.6841.[1] Therefore, the average of the thirty selected ordinates, multiplied by 10.6841 gives the X coefficient of the source under examination. In practice, the separate steps of averaging and of multiplying by 10.6841 are combined into a single multiplication by the factor listed in the table, i.e., $10.6841/30=0.3561$. The magnitude of this multiplying factor, while independent of the energy units themselves, is influenced by the width of the unit wave band specified. The present factors are based upon a unit wave band of 10 millimicrons. For 5 millimicrons, for instance, the multiplying factors would all be twice the values shown in Table I.

Once the trichromatic coefficients X, Y and Z are known, the tristimulus values are readily obtained, and from these, the color temperature of the source can be calculated, using the method of Judd.[2] The inaccuracies of specification by color temperature applied to colors which are appreciably different from that of a black body at any temperature should, of course, be borne in mind in this connection.

The candlepower of the source can also be calculated from the trichromatic coefficient, Y. By taking into account the distance, d, in cm between the light source and the surface at which irradiance was measured, it is found that

$$\text{Candlepower}=0.000621\,Yd^2,$$

where Y equals the tristimulus value determined by weighting energy units expressed in microwatts per square centimeter. This calculation is often of considerable value in checking measured values of candlepower of a source whose color is markedly different from that of the incandescent standard customarily used in a visual comparison method.

Only thirty selected ordinates have been determined by the writers, this number being quite sufficient for calculations with the relatively smooth radiation curves of the various carbon arcs, whose color evaluation made this work desirable. For the various types of mercury arcs, for fluorescent lamps, and other light sources whose spectra show strong isolated lines, the 30-selected-ordinate method should be avoided because it is inaccurate in such cases, and indeed it is doubtful whether the selected-ordinate method regardless of the number of ordinates used is suited to the evaluation of such sources.

[1] Based upon the tristimulus values for spectrum colors listed in Table XII, pp. 35–38 incl., in the *Handbook of Colorimetry*. If the abbreviated Table I, p. 7 of this book, is used in the calculation, a value of 10.6666 is obtained.

[2] "Estimation of Chromaticity Differences and Nearest Color Temperature on the Standard 1931 I. C. I. Colorimetric Coordinate System," by D. B. Judd, J. O. S. A. **26**, 421–426 (1936).

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